Polychromatic Reconstruction for Talbot-Lau X-ray Tomography

ERLANGEN CENTRE FOR ASTROPARTICLE PHYSICS

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FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG

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Outline

- Motivation
- Tomography
- State of the art
- Contributions
- Results
- Outlook

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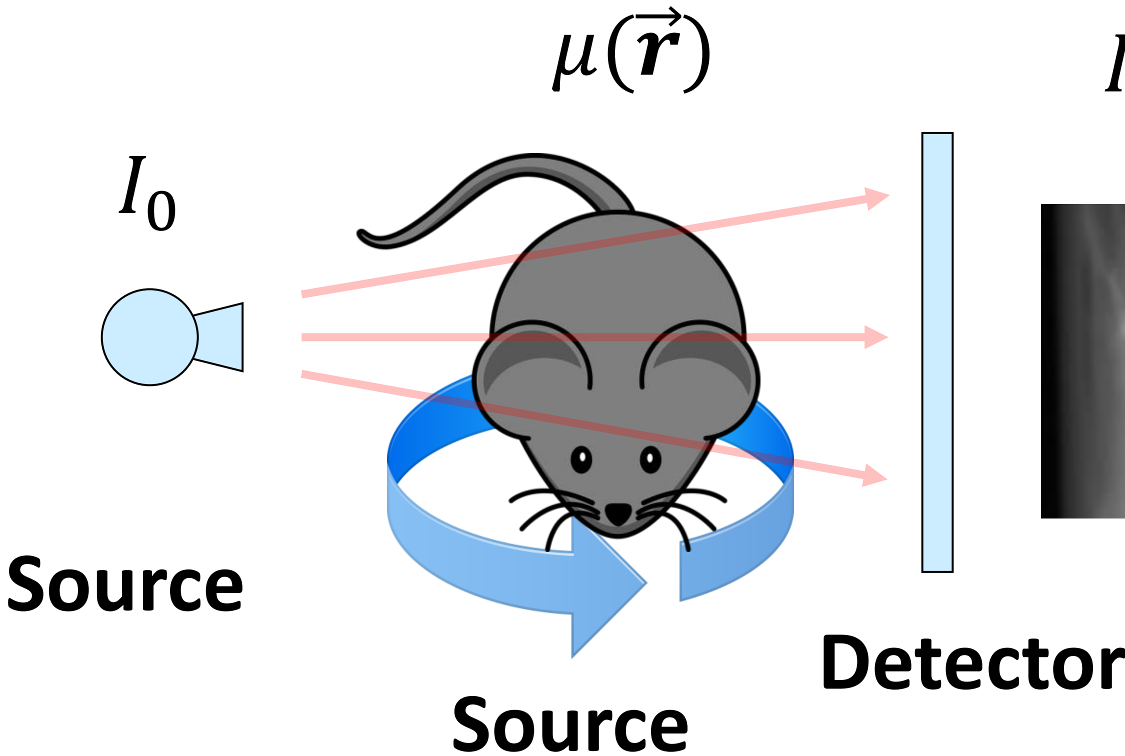
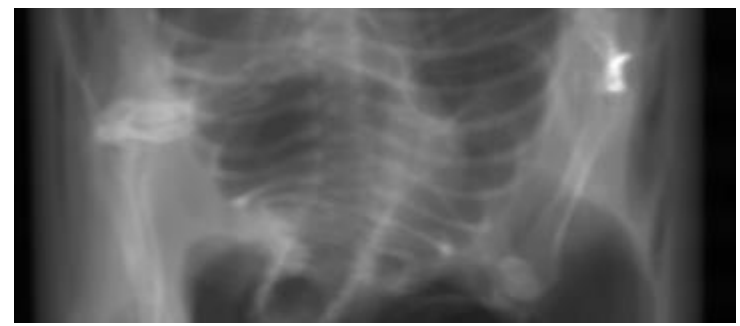
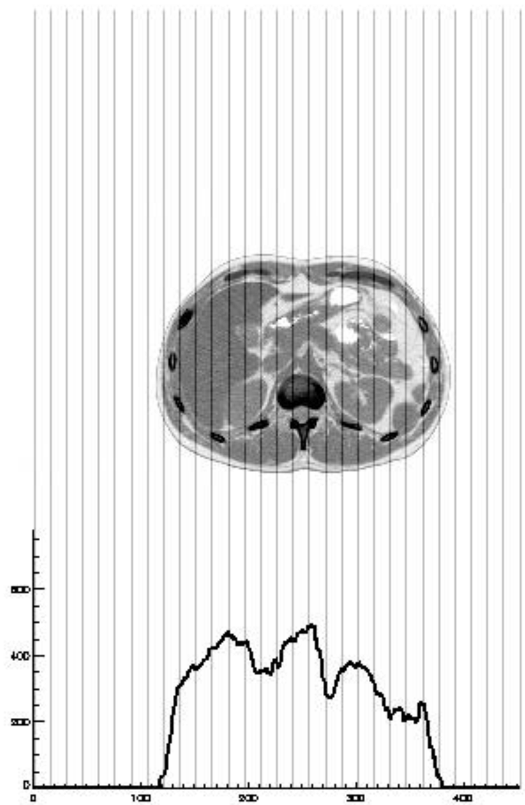
Beer-Lambert Law

$$I = I_0 e^{-\int \mu(\vec{\boldsymbol{r}}) d\vec{\boldsymbol{r}}}$$



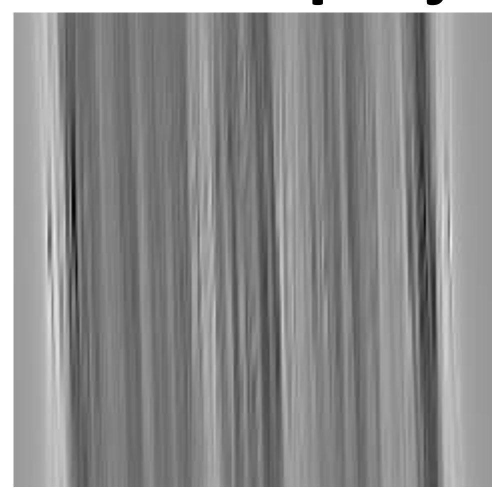


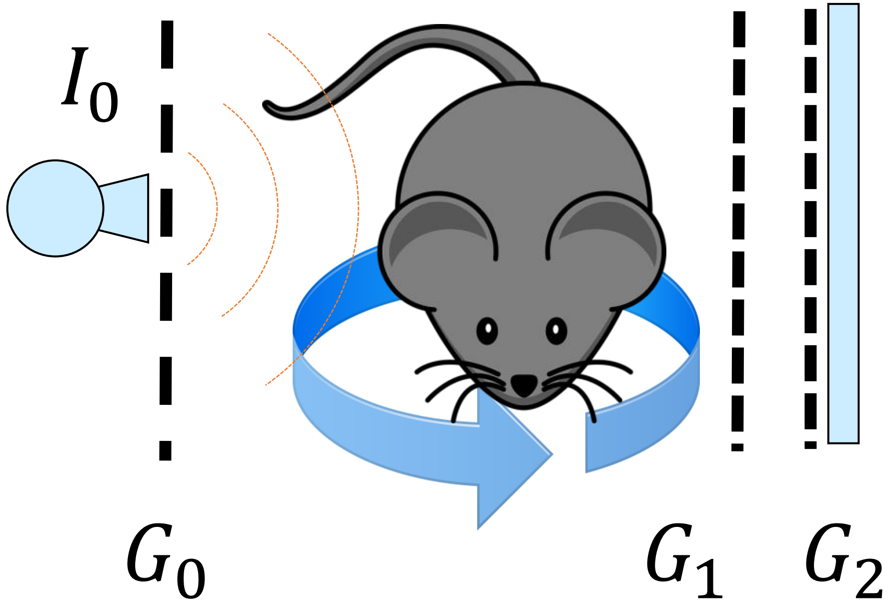
Projection



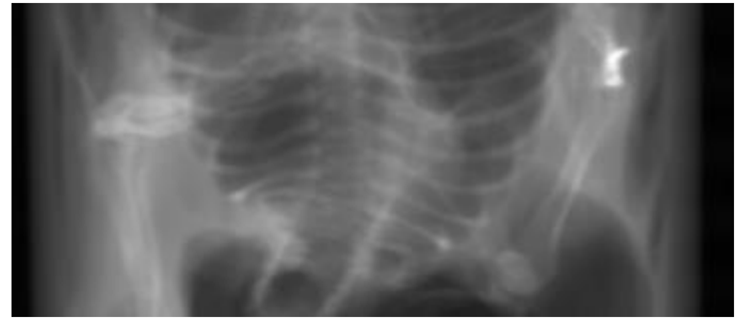
Radon transform

Filtered back projection

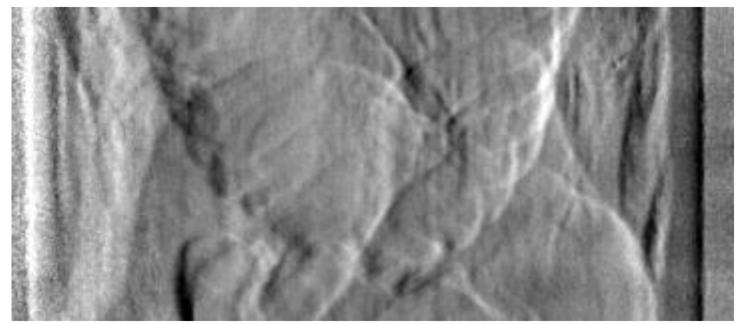




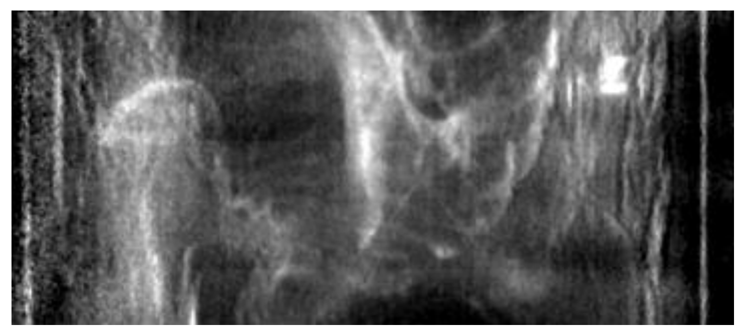
Attenuation

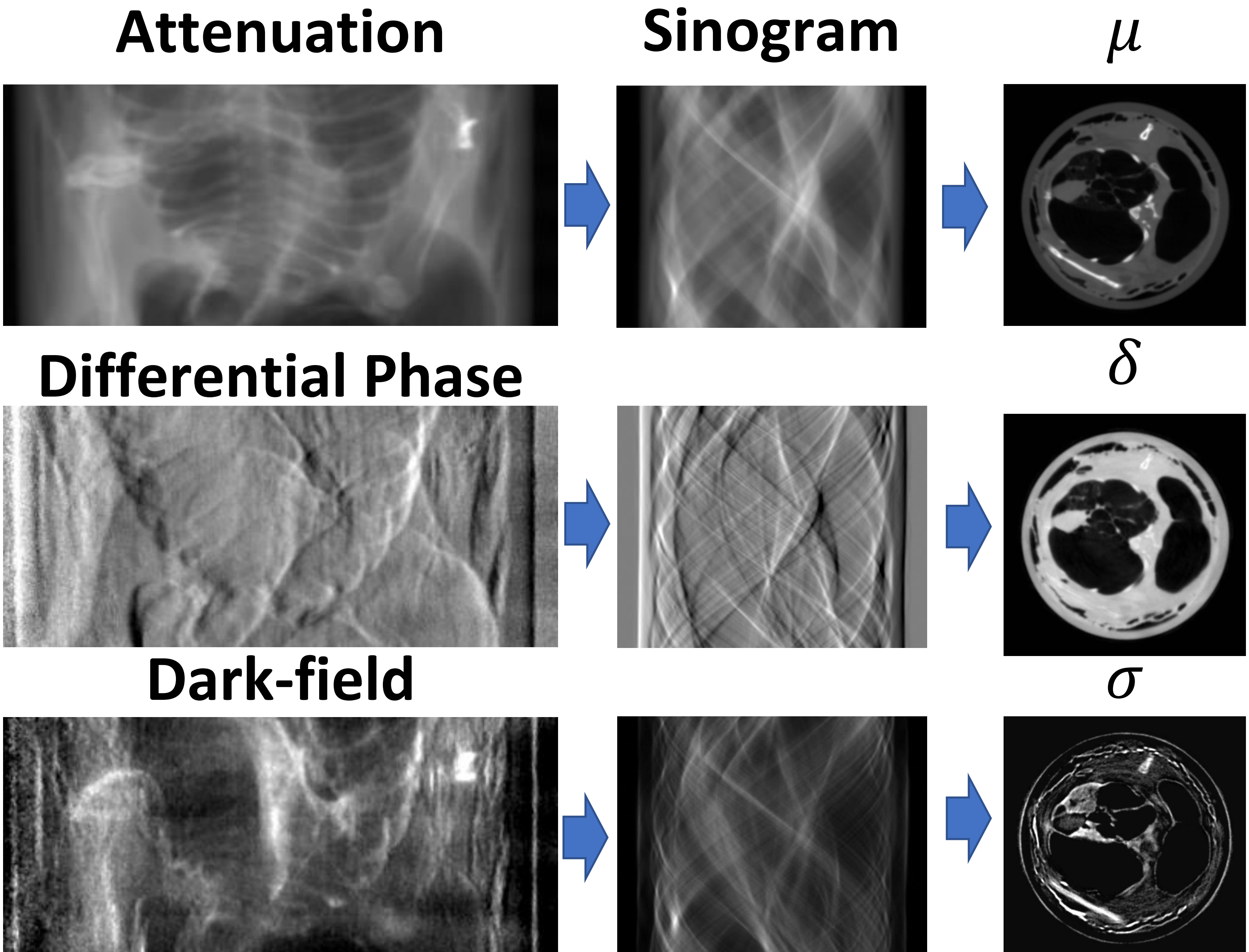


Differential Phase

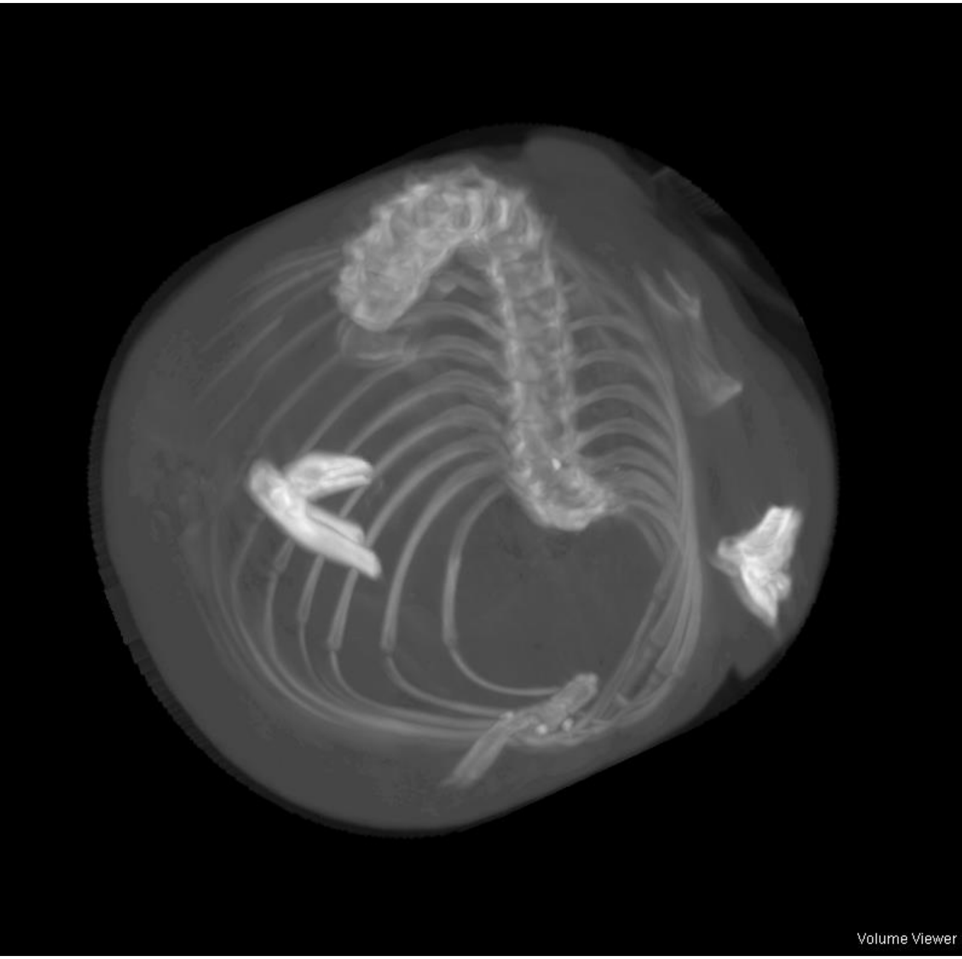


Dark-field

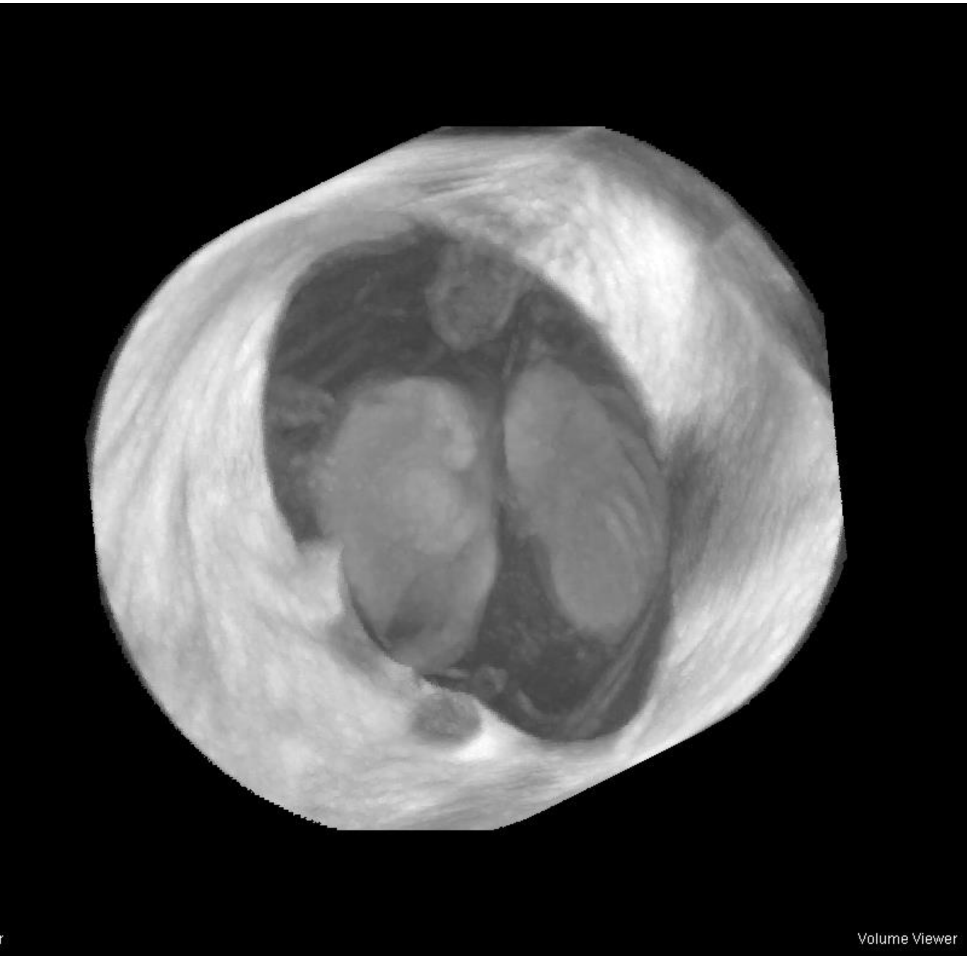



Attenuation
Sinogram
μ
Differential Phase
δ
Dark-field
σ

Attenuation



Scatter



Iterative Reconstruction In Computed Tomography

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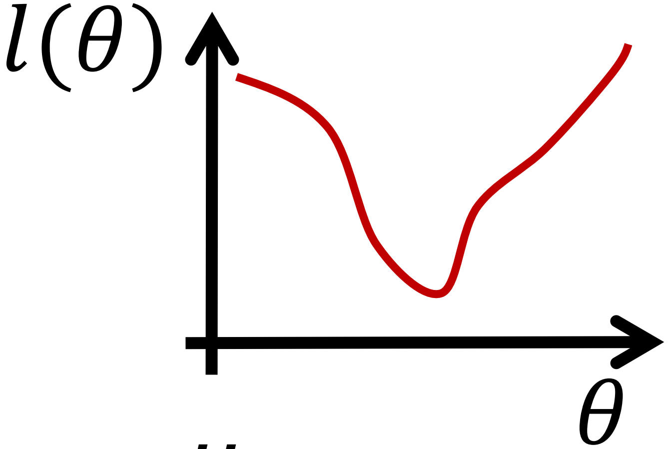
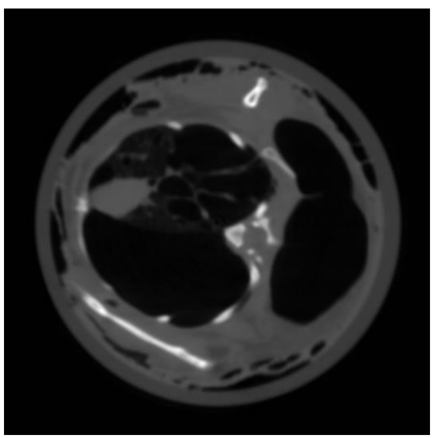
Iterative reconstruction



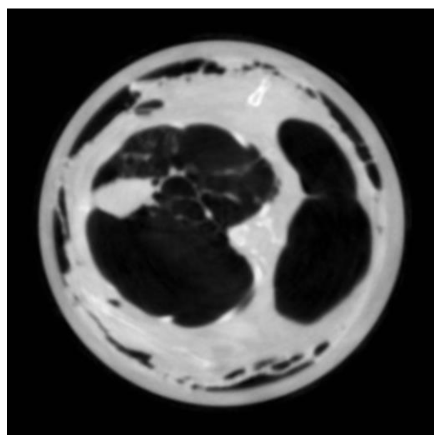
Minimize cost function

$$\theta = \{\mu, \delta, \sigma\}$$

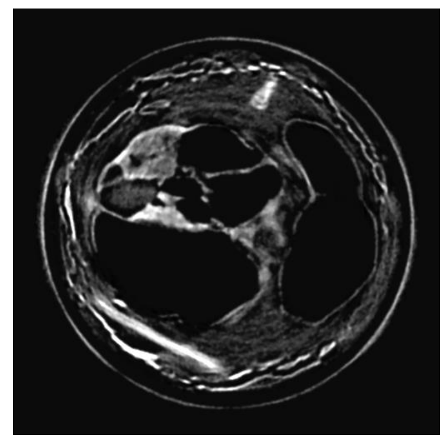
μ



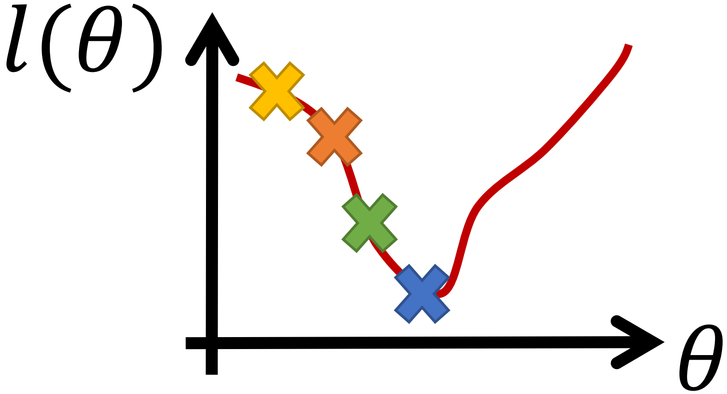
δ



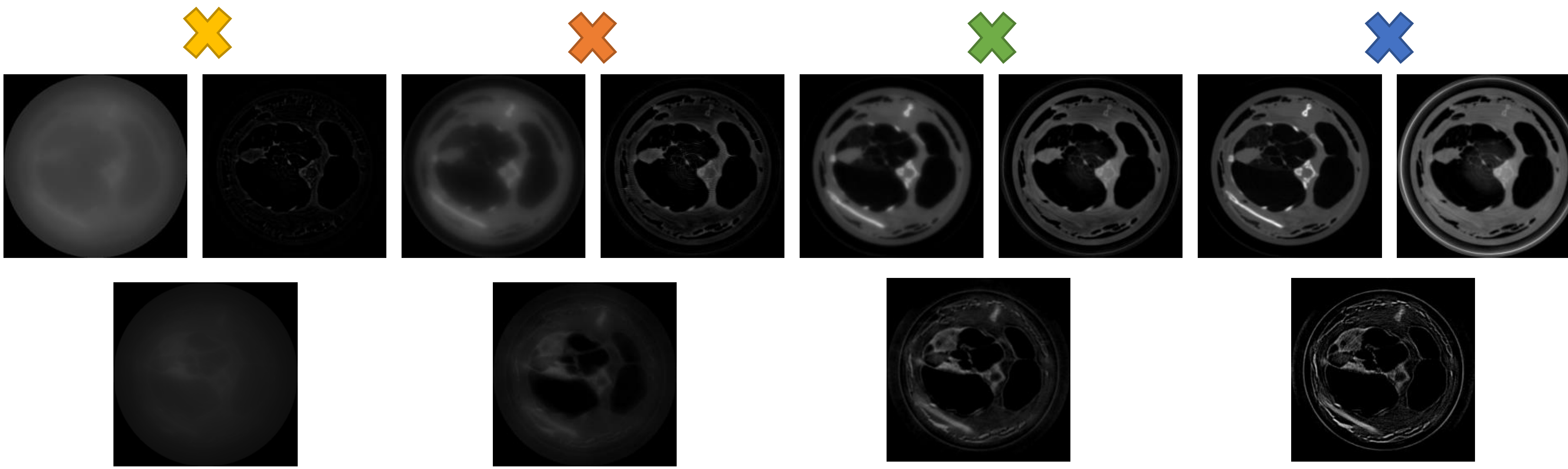
σ



Iterative reconstruction



Minimize cost function



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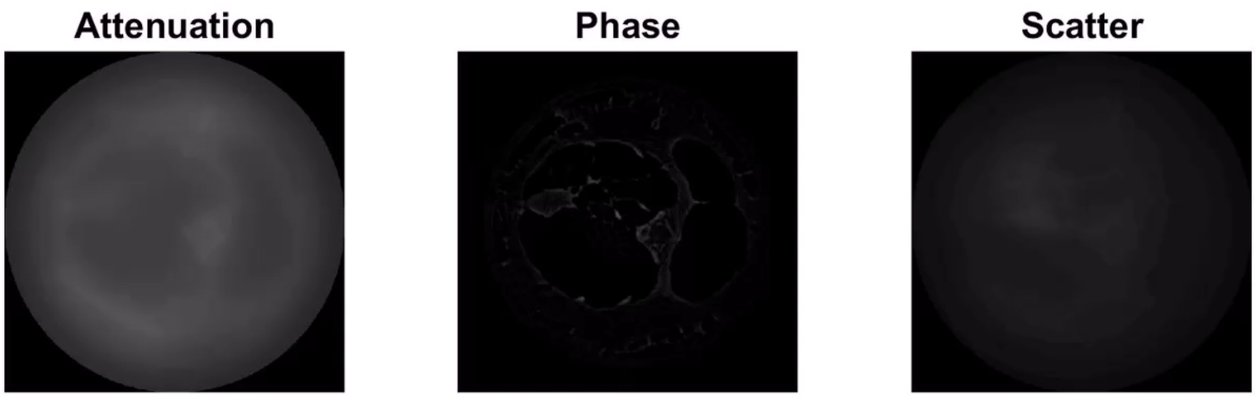

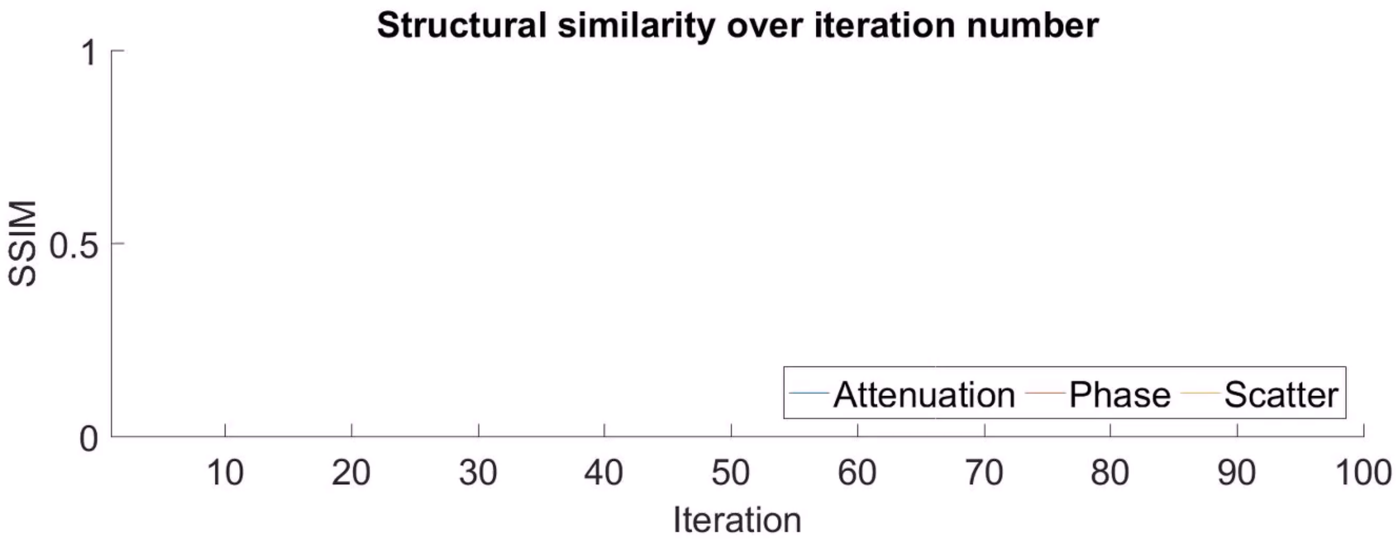

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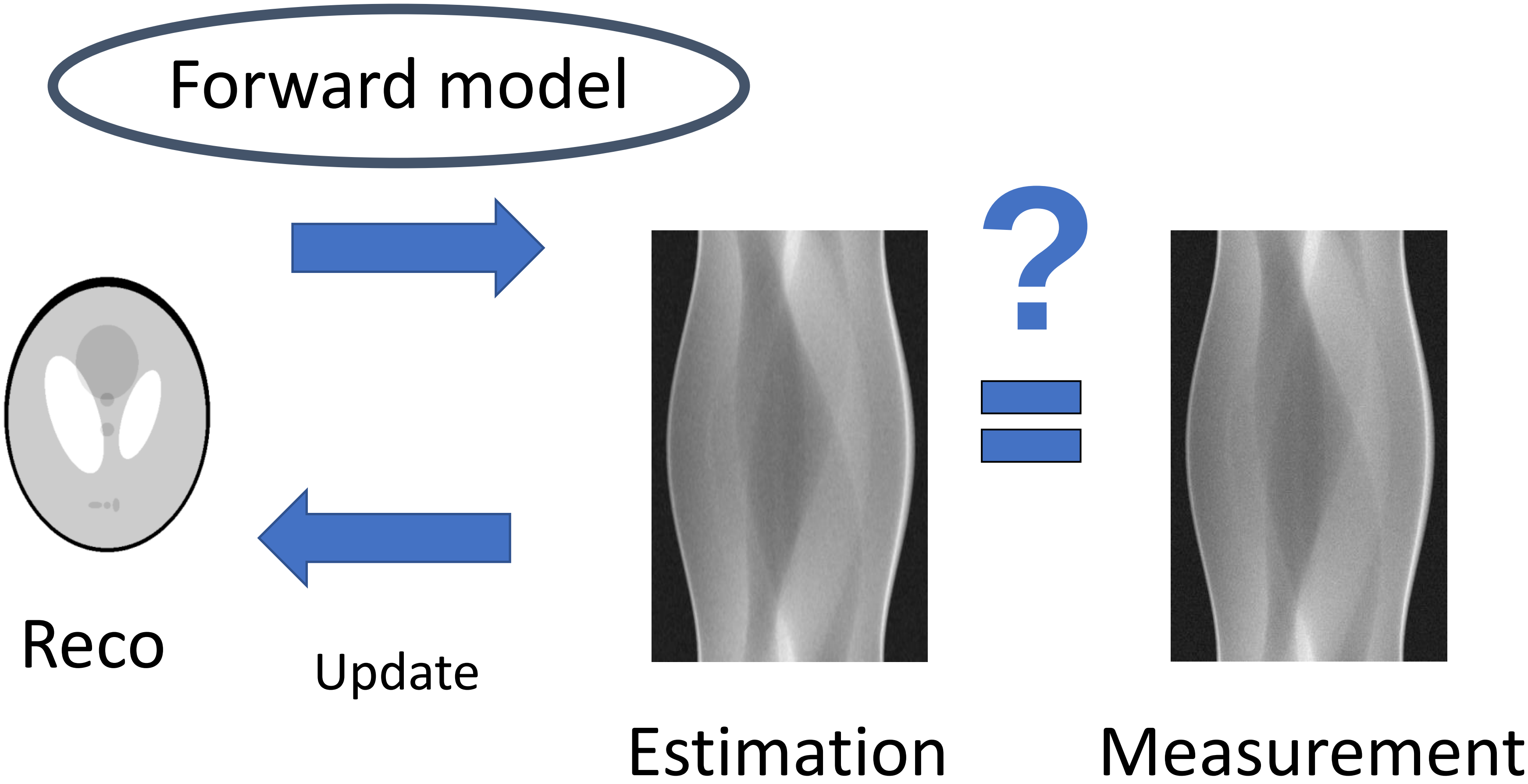
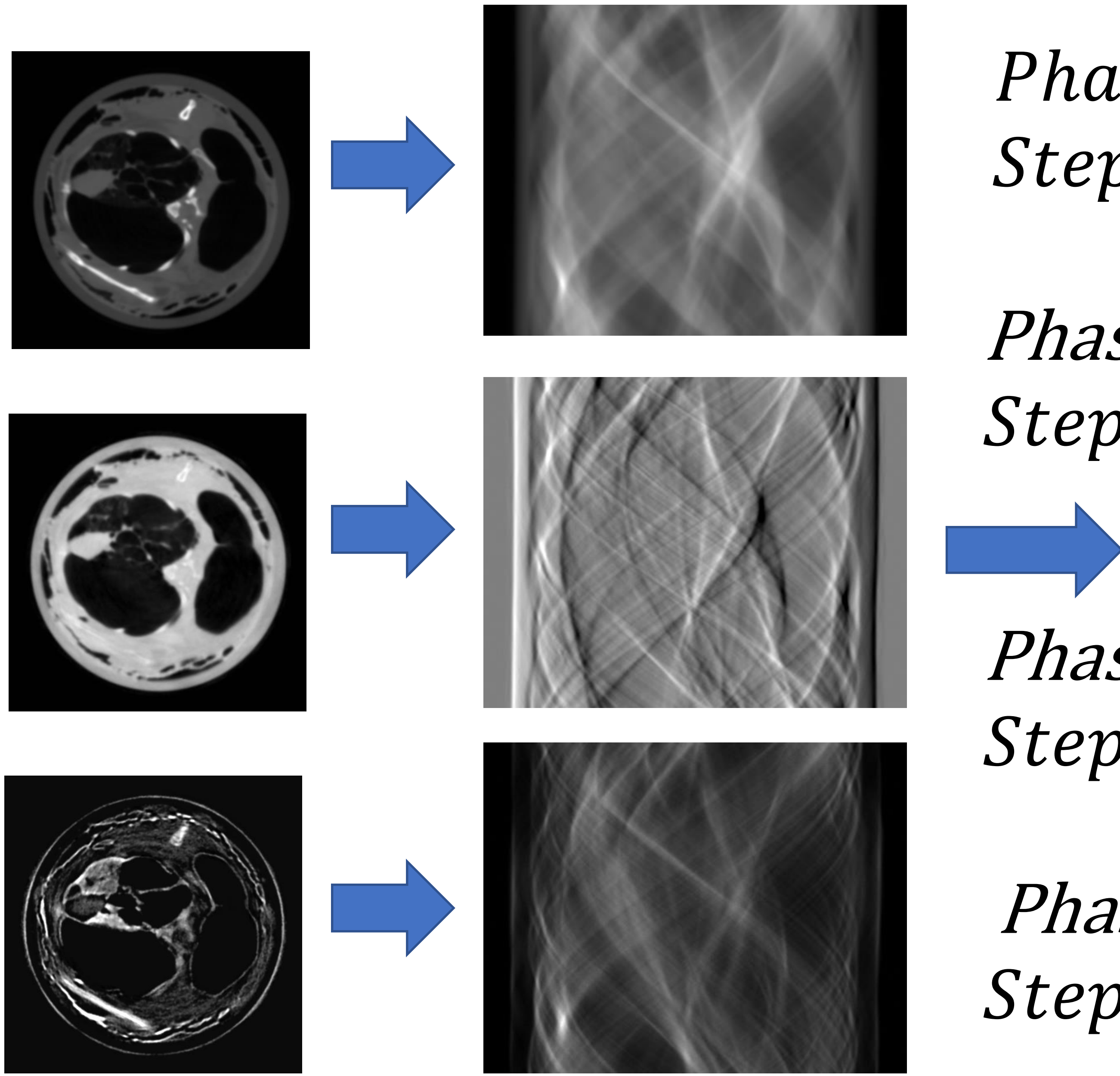
Iterative reconstruction



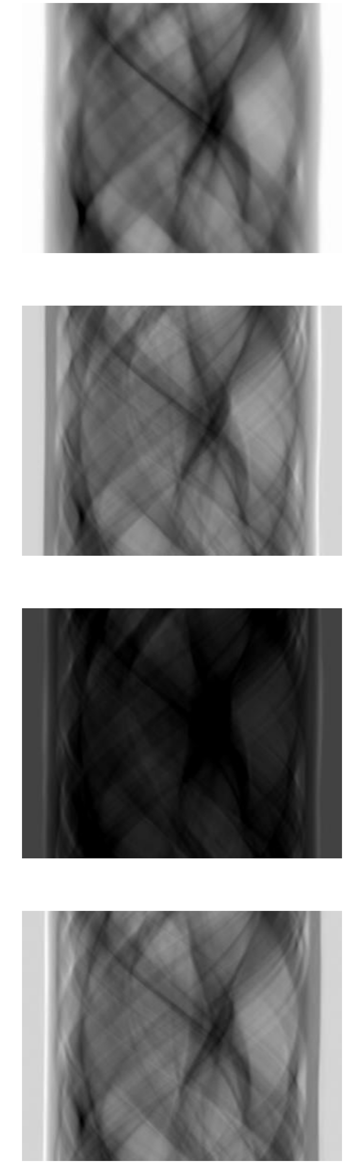


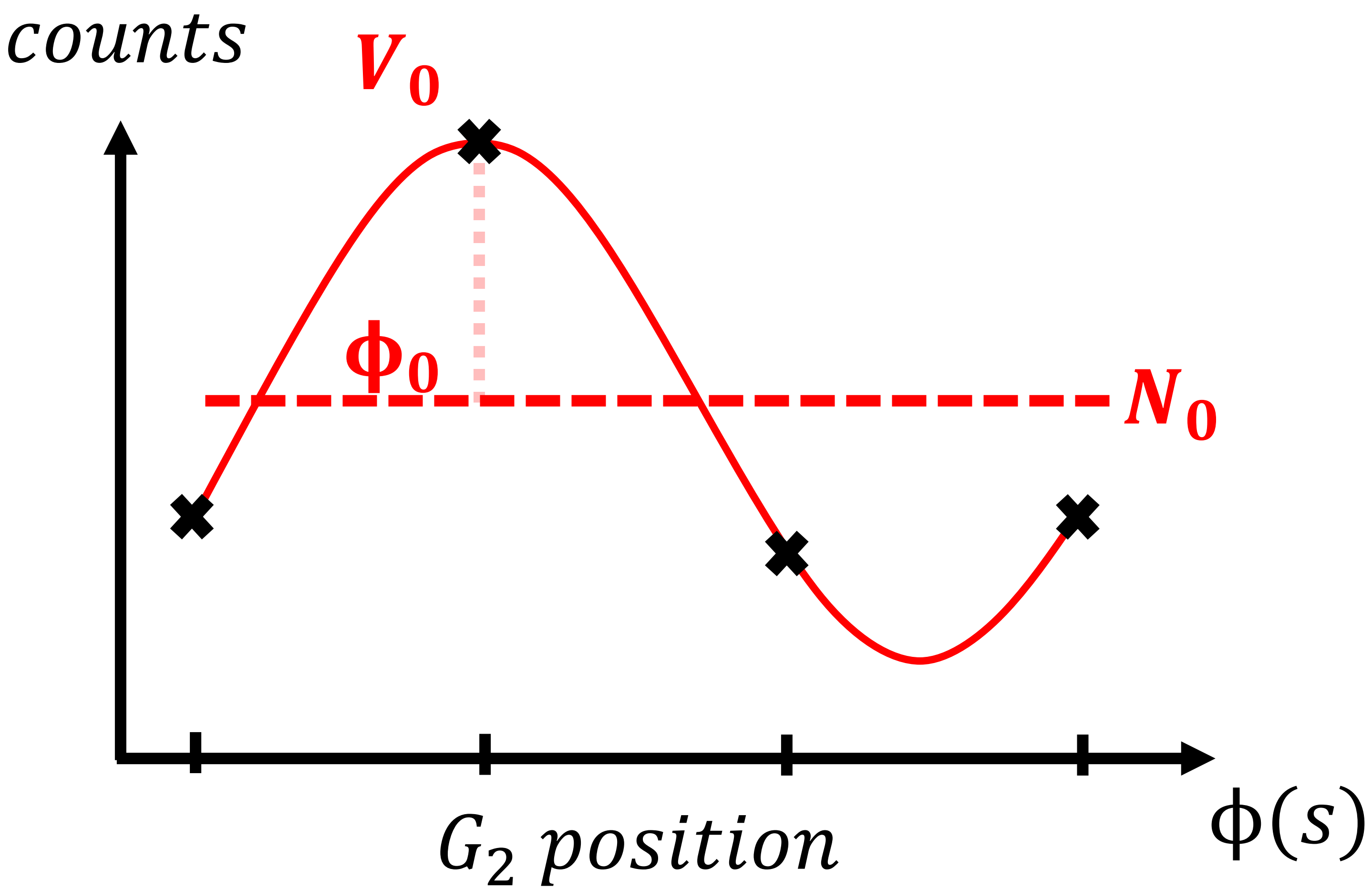
Phase
Step 1

Phase
Step 2

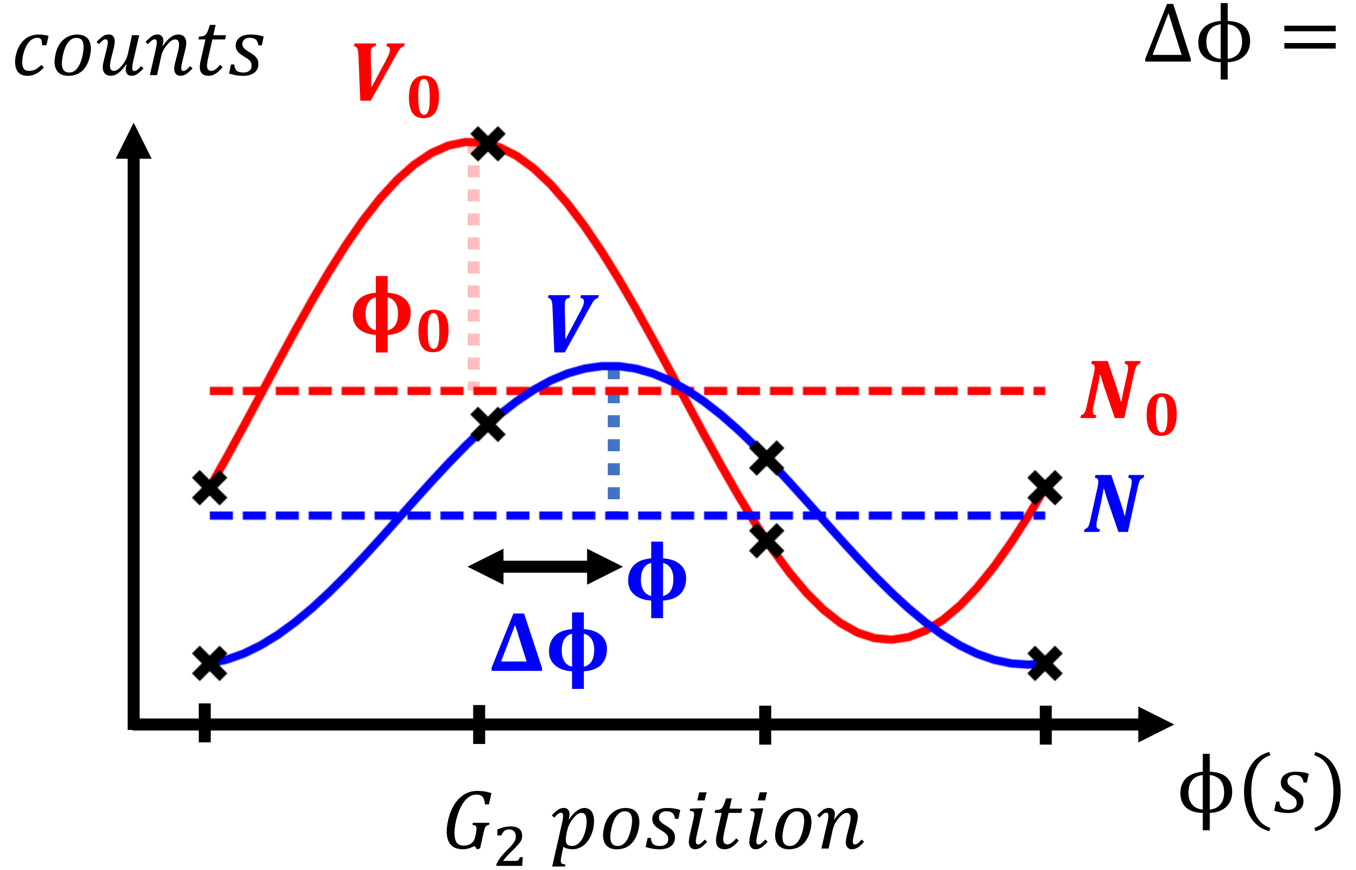
Phase
Step 3

Phase
Step 4





$$N_s = N_0 \cdot (1 + V_0 \cos \phi_0 + \phi_s)$$



$$\Delta\phi = \phi_0 - \phi$$

$$T = \frac{N}{N_0}$$

$$D = \frac{V}{V_0}$$

$$N_s = N_0 T \cdot (1 + D V_0 \cos \Delta\phi + \phi_0 + \phi_s)$$

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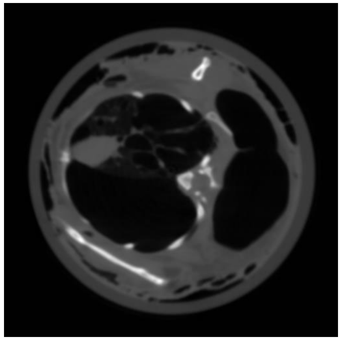
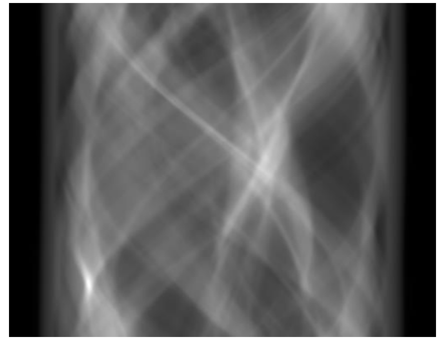
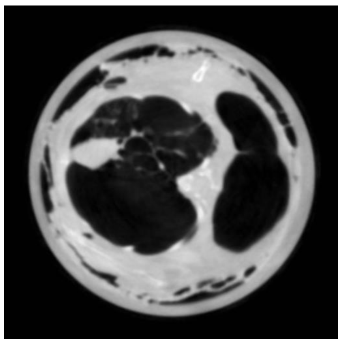
μ

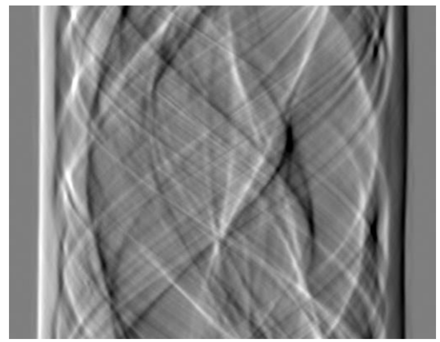




$$T = e^{-\int \mu(\vec{r})d\vec{r}}$$

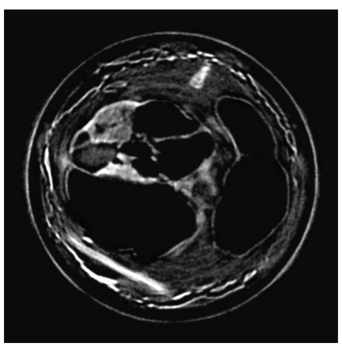
δ

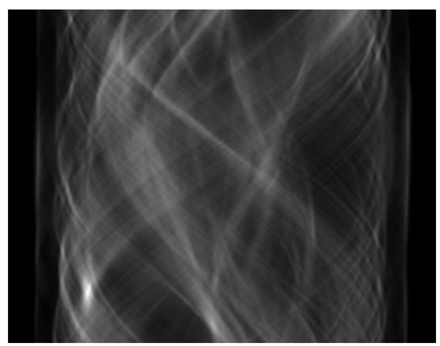




$$\Delta\phi = \frac{d}{dx}\int \delta(\vec{r})\, d\vec{r}$$

σ





$$D = e^{-\int \sigma(\vec{r})d\vec{r}}$$

State of the art

- Andre Ritter et al. (2013, ECAP)
- Bernhard Brendel et al. (2016, Philips)
- Andre Ritter et al. (2016, ECAP)
- Andreas Wolf (2016, ECAP)

Ritter et al. (2013)

μ δ σ

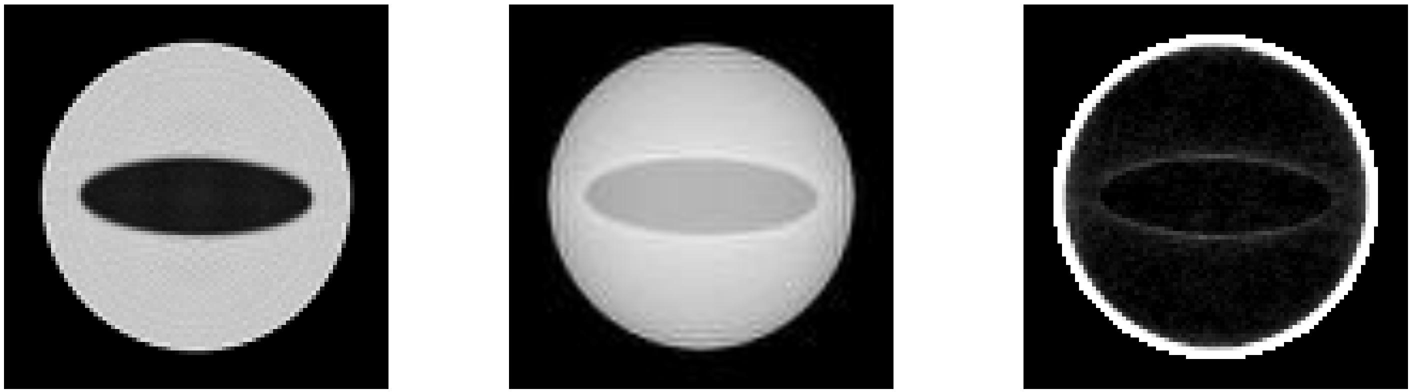


Image resolution: 90×90

Simulated data (CXI)

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Brendel et al. (2016)

μ δ σ

Synchrotron data

Reconstruction with interlaced acquisition

Regularization term added

Andreas Wolf (ECAP, Master thesis)

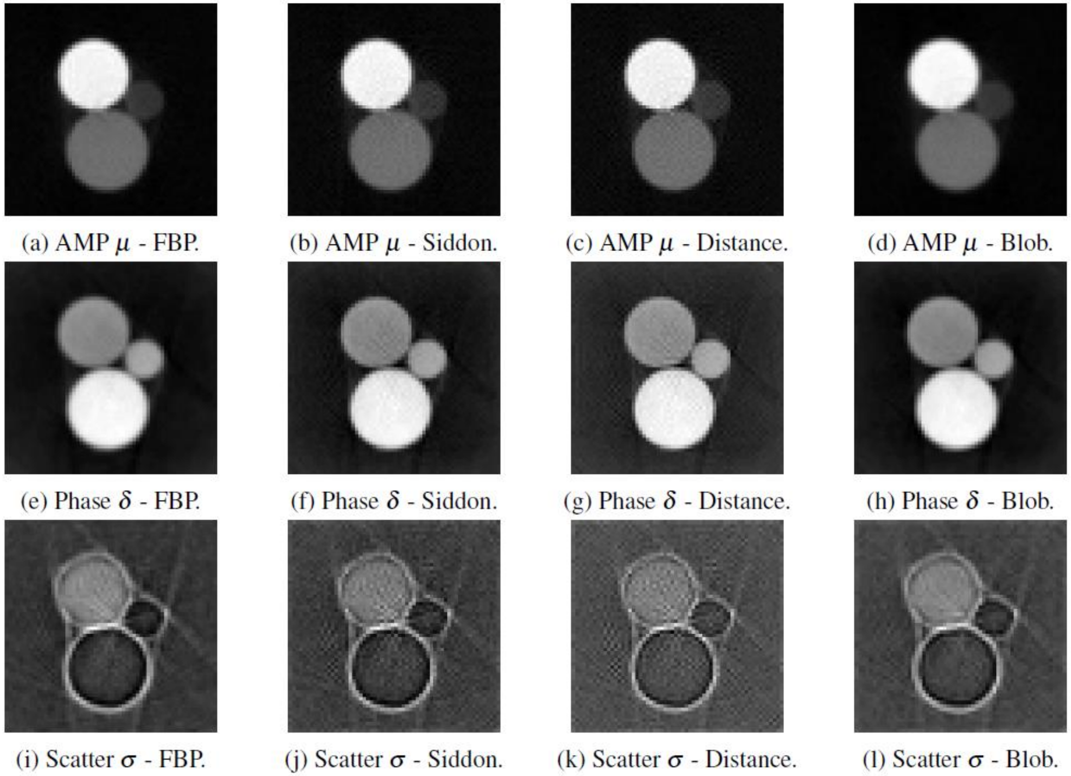


Image resolution: 60×60

Ritter et al. (2016)

μ δ σ

Image resolution: 51×51

Real data of biological sample with conventional X-ray tube

Reconstruction Framework of this thesis

μ δ σ

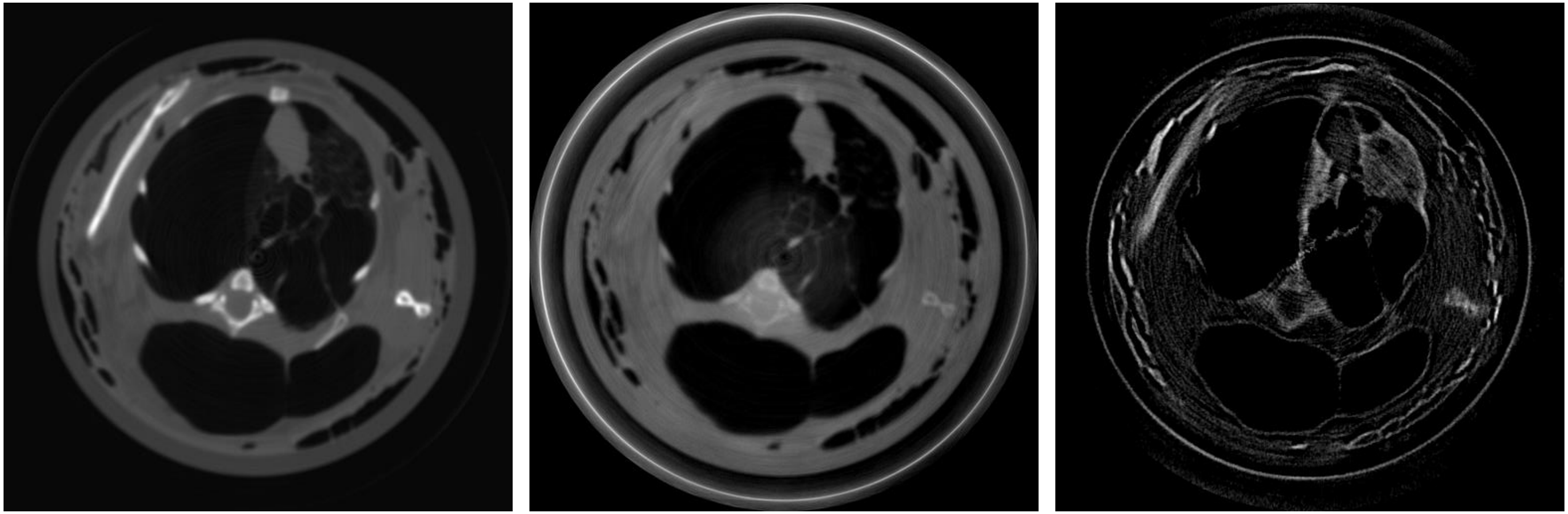


Image resolution: 512×512

Contributions

- Development of reconstruction framework
    - Polychromatic artifacts
    - Enhanced optimization algorithm
- Development of (pre-)processing methods
- Numerical analysis of the reconstruction algorithm
- Planning and execution of tomographic measurements to evaluate the proposed algorithms

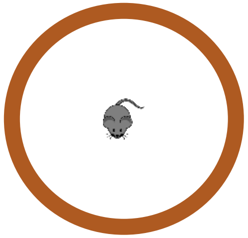
Scalable to medical imaging?

Beam hardening

Noise

Stability ✓

Gratings



Energy dependence of

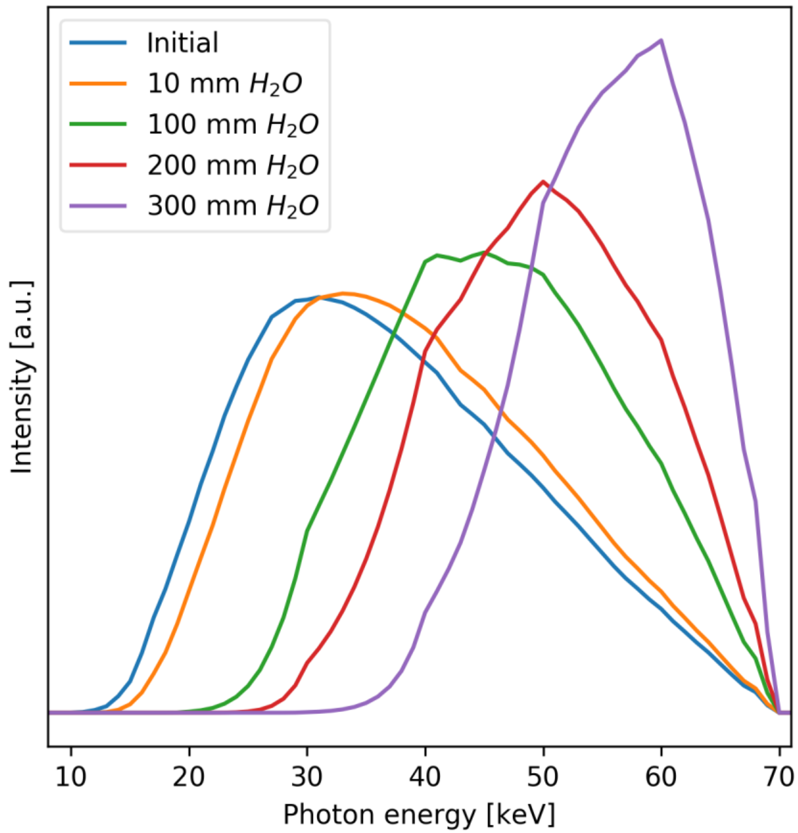
Material

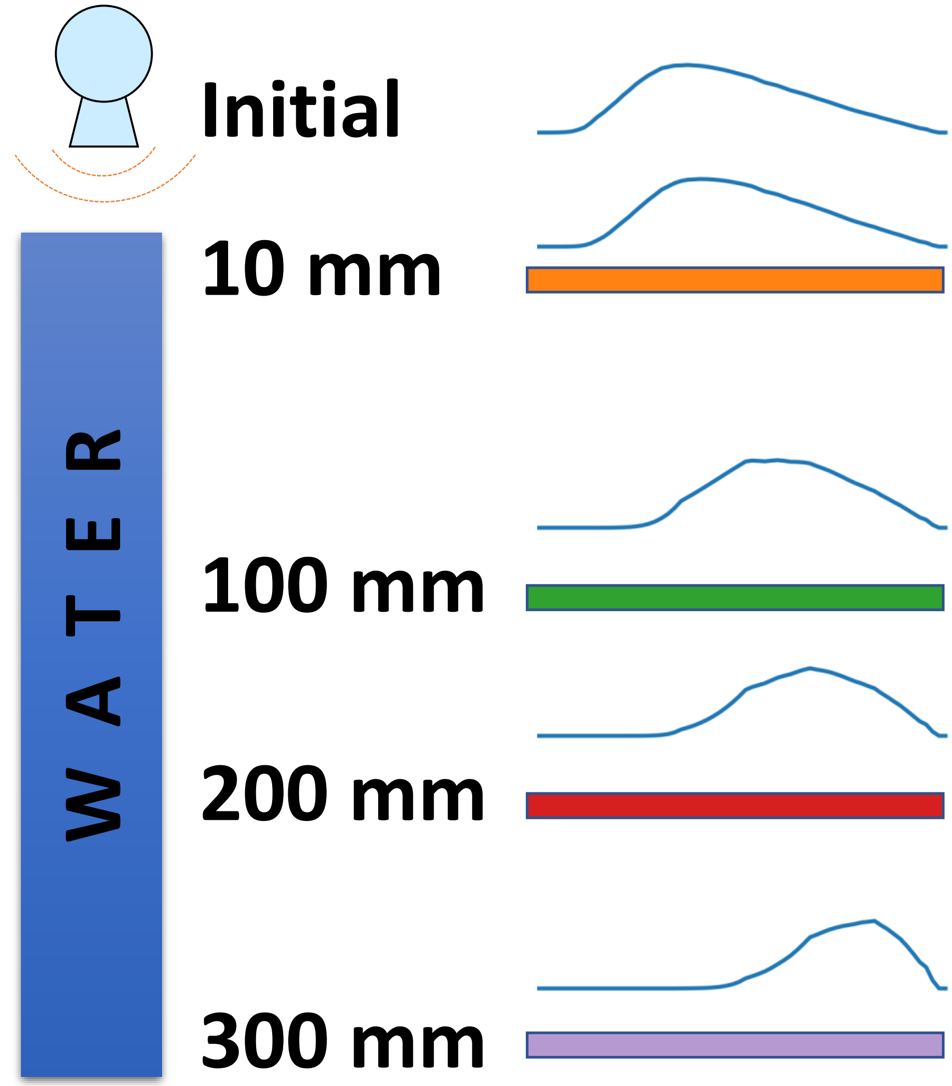
$$\mu(E), \delta(E), \sigma(E)$$

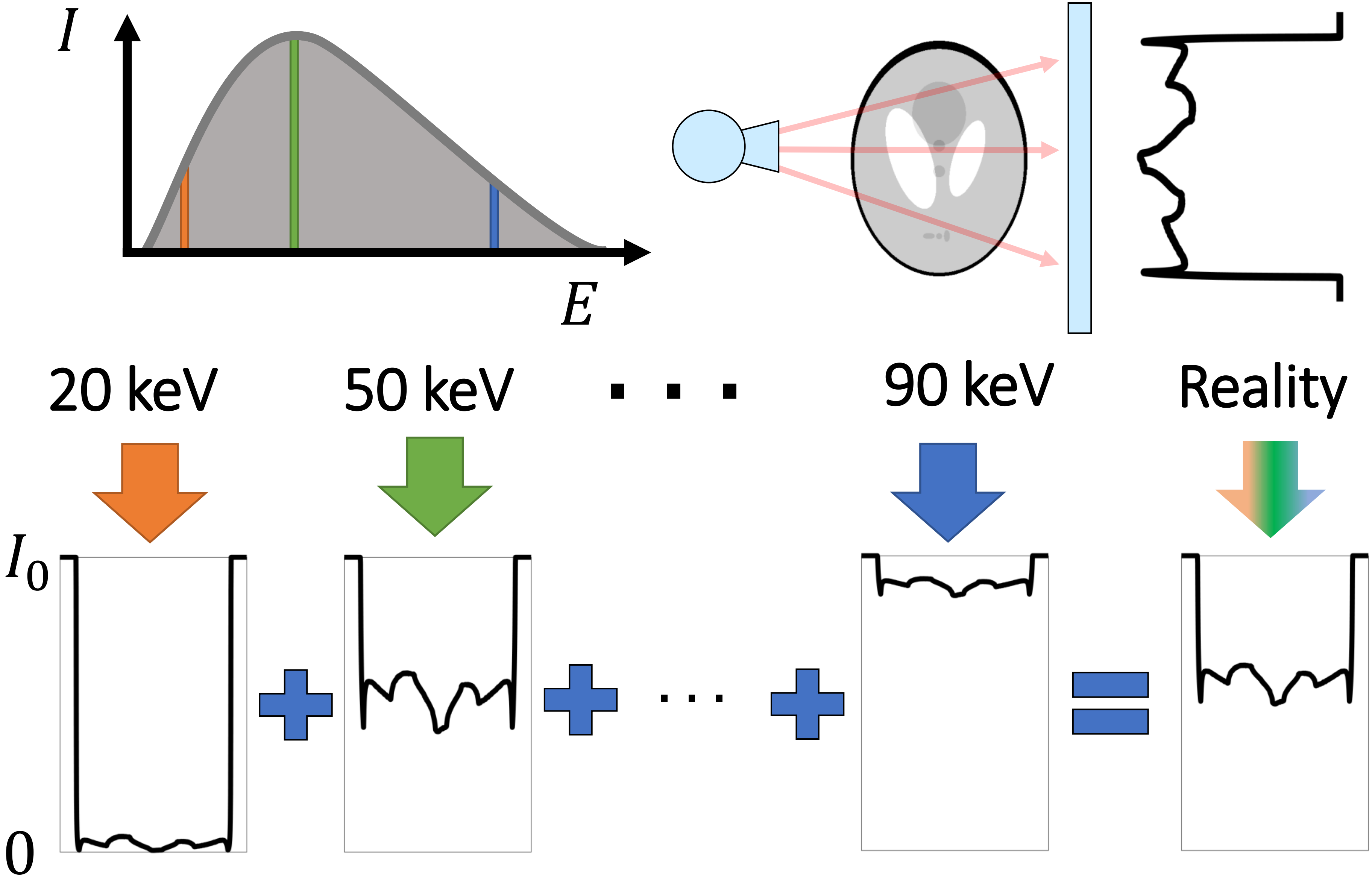
Interferometer

$$N_0(E), \phi_0(E), V_0(E)$$

Beam hardening

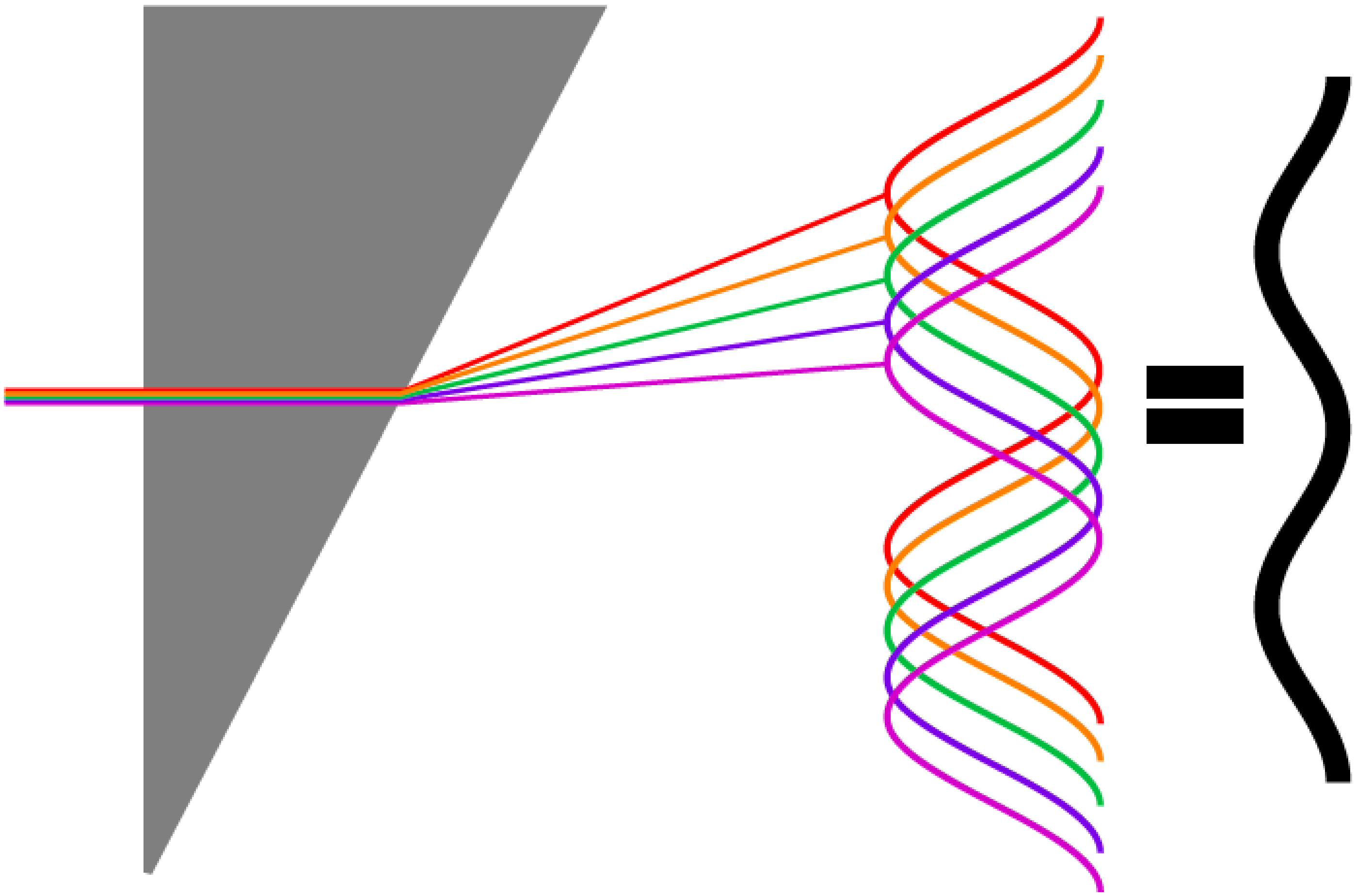






I
E
20 keV
50 keV
. . .
90 keV
Reality
I_0
0
. . .

Dispersion



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Energy dependence of

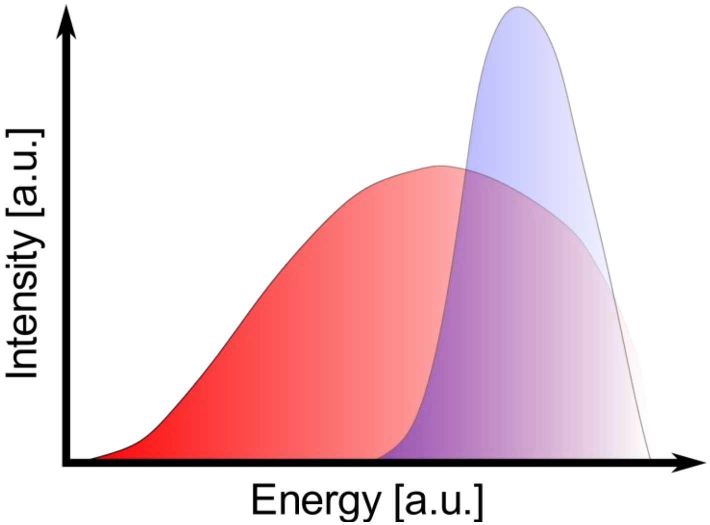
Material

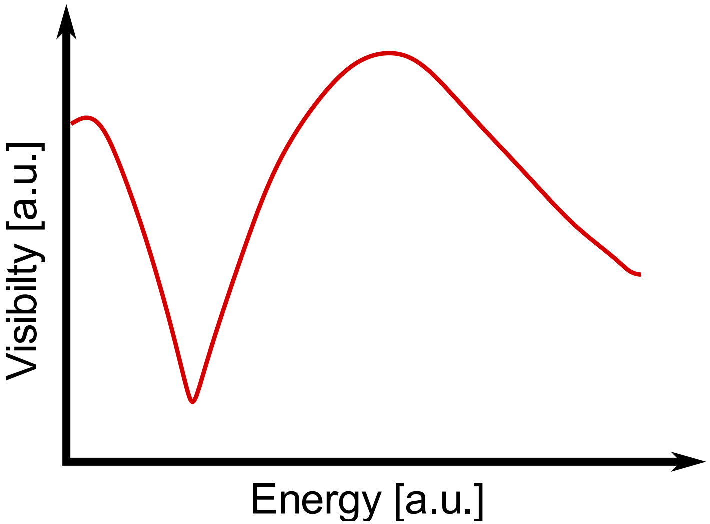
$$\mu(E), \delta(E), \sigma(E)$$

Interferometer

$$N_0(E), \phi_0(E), V_0(E)$$

Dark-field due to beam hardening





$$\bar{N} = \int N(E) dE$$

$$\bar{V} = \frac{\int N(E) \cdot V(E)\, dE}{\int N(E)\, dE}$$

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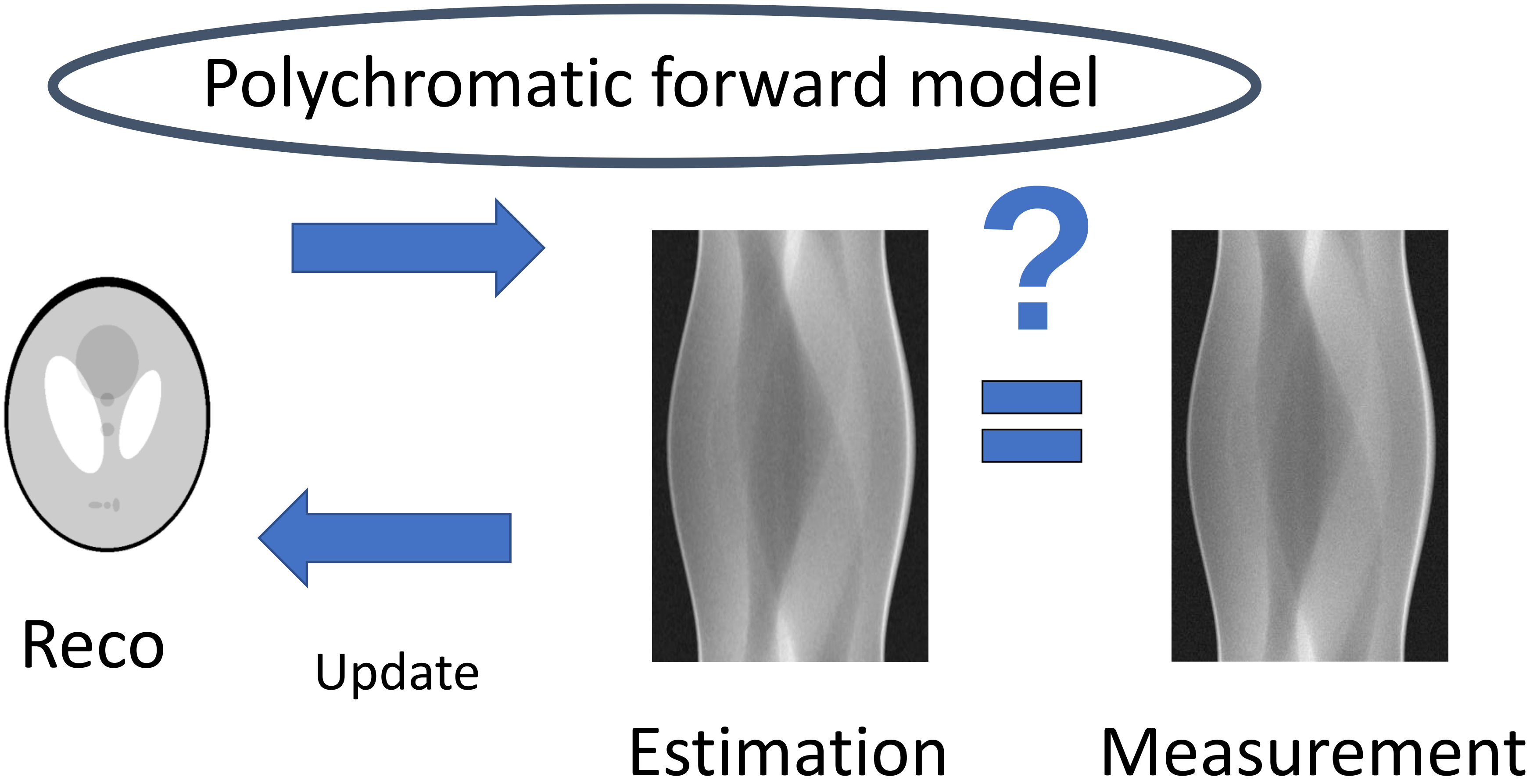
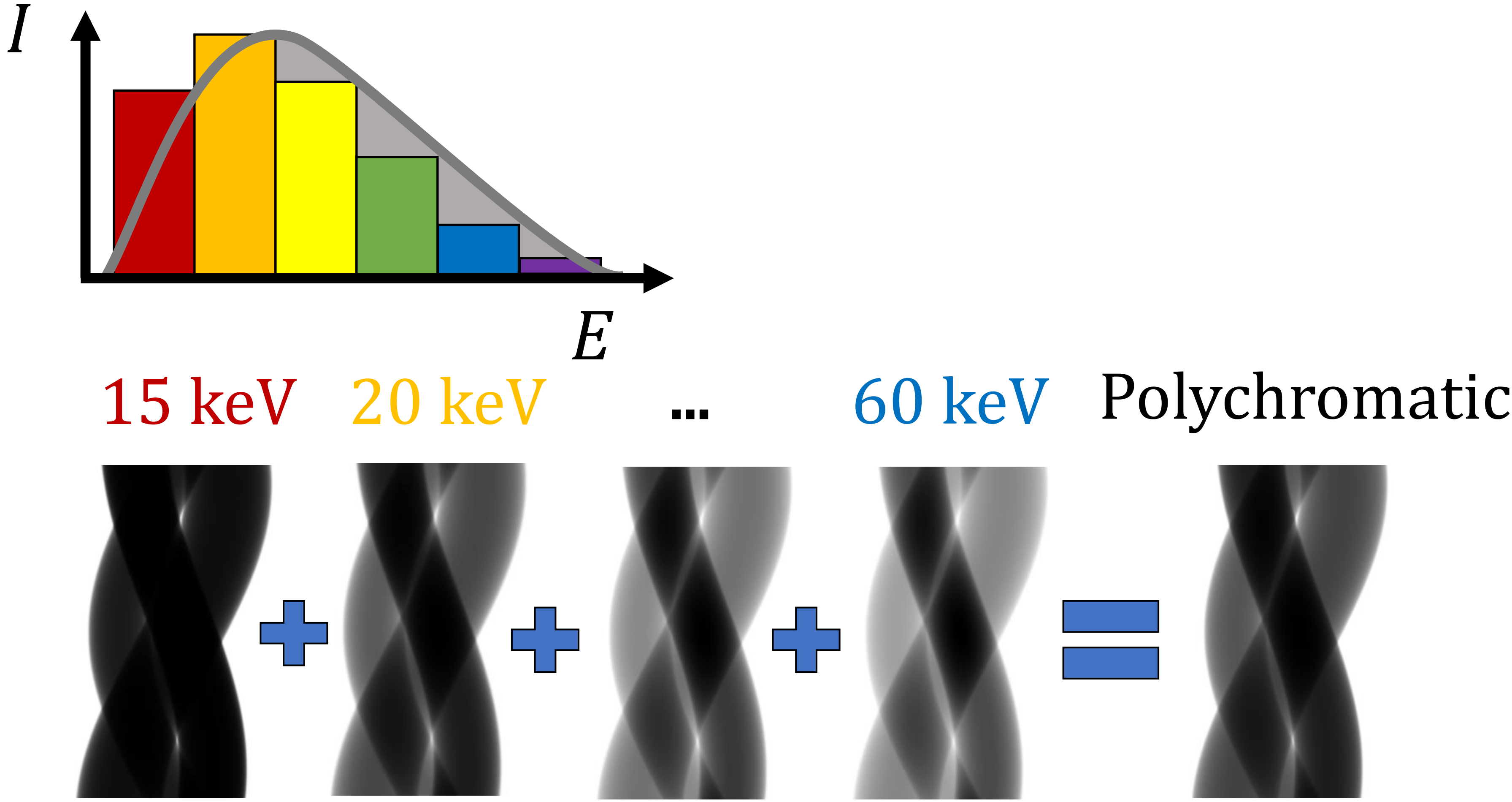
How to deal with beam hardening?

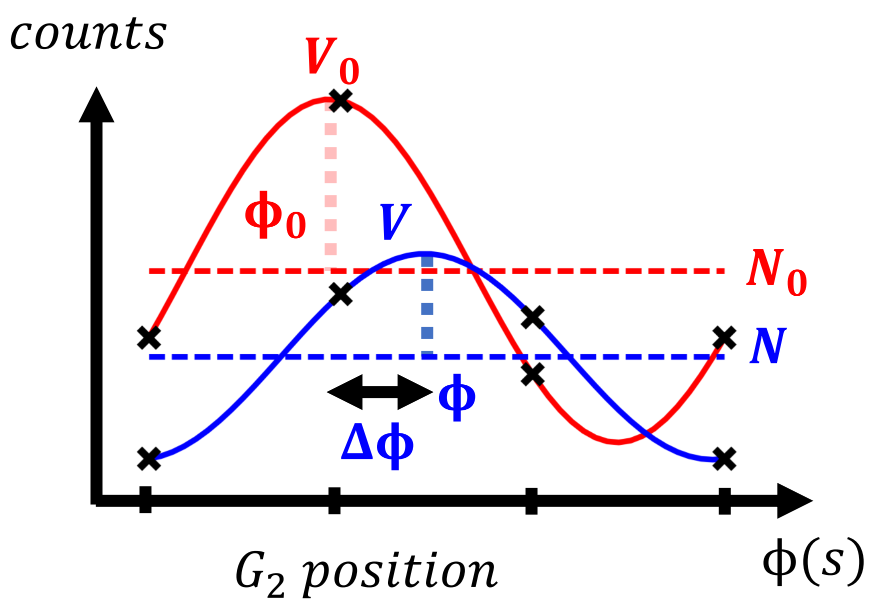
Iterative reconstruction



From monochromatic to polychromatic

Polychromatic Forward Model





$$\Delta\phi(E) = \phi_0(E) - \phi(E)$$

$$T(E) = \frac{N(E)}{N_0(E)}$$

$$D(E) = \frac{V(E)}{V_0(E)}$$

$$N_s = \int dE\, N_0(\boldsymbol{E})\boldsymbol{T}(\boldsymbol{E}) \cdot$$

$$(1 + \boldsymbol{D}(\boldsymbol{E})V_0(E)\cos[\boldsymbol{\Delta\phi}(\mathbf{E}) + \phi_0(E) + \phi_s])$$

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Polychromatic Forward Model

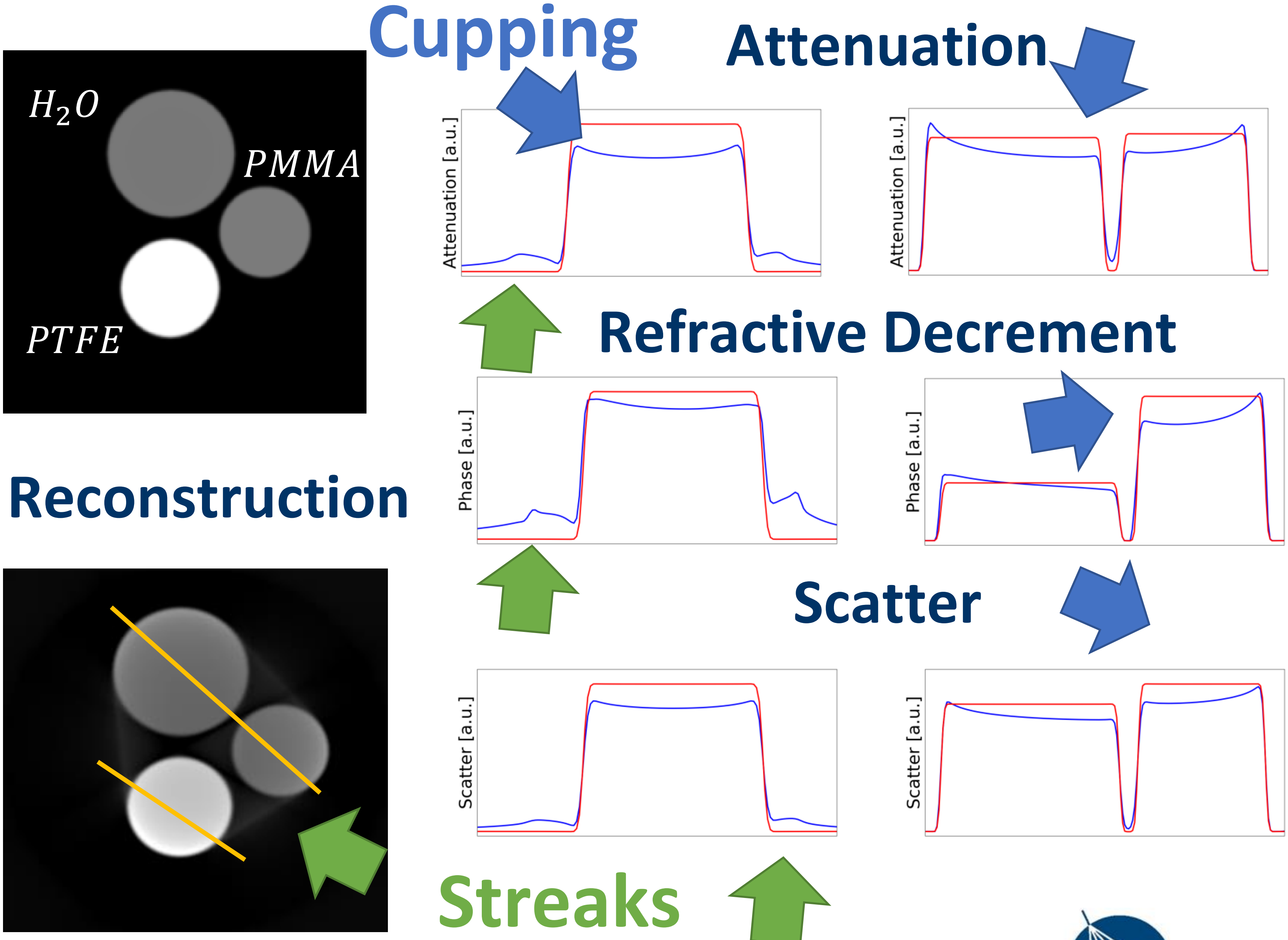
$$\mu(E) = \mu(E_0) \cdot \left(\frac{E}{E_0}\right)^{c_\mu = -3}$$

$$\delta(E) = \delta(E_0) \cdot \left(\frac{E}{E_0}\right)^{c_\delta = -2}$$

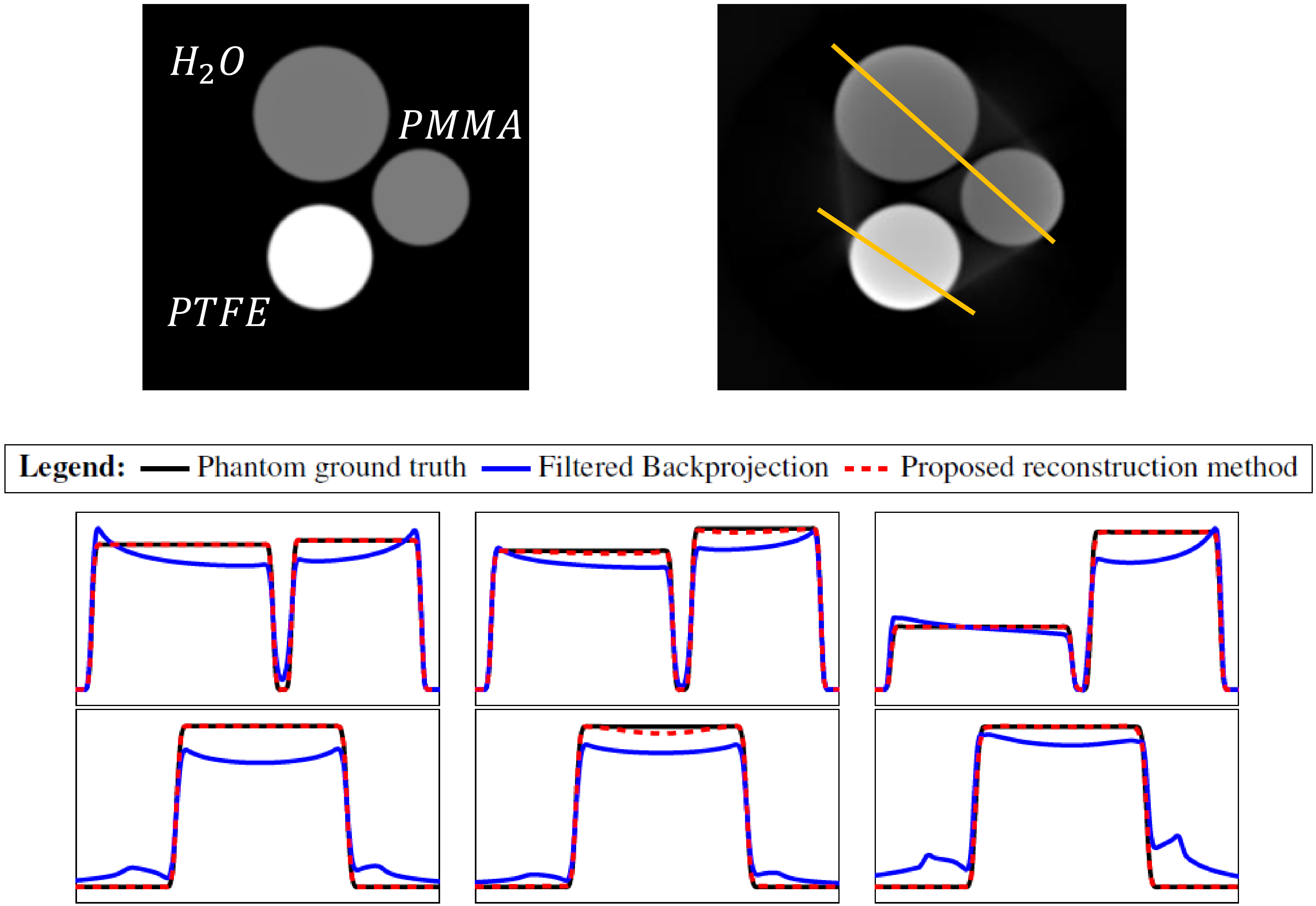
$$\sigma(E) = \sigma(E_0) \cdot \left(\frac{E}{E_0}\right)^{c_\sigma = -2}$$

Results

Simulation data



Cupping
Attenuation
H_2O
$PMMA$
$PTFE$
Attenuation [a.u.]
Attenuation [a.u.]
Refractive Decrement
Reconstruction
Phase [a.u.]
Phase [a.u.]
Scatter
Scatter [a.u.]
Scatter [a.u.]
Streaks



(a) Attenuation (b) Refractive decrement (c) Scatter

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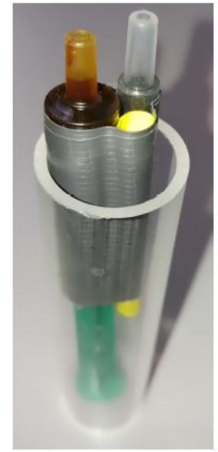
Real data

Specimen

(a)

(b)

(c)



(d)

(e)

(f)

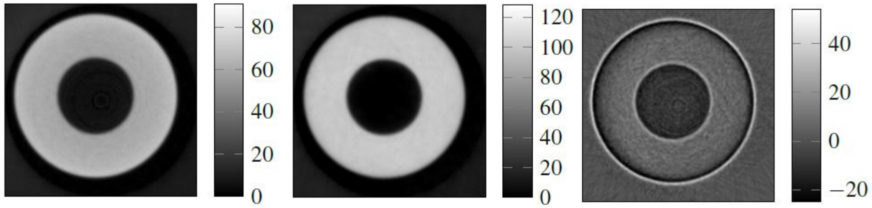
(g)

(h)

(i)

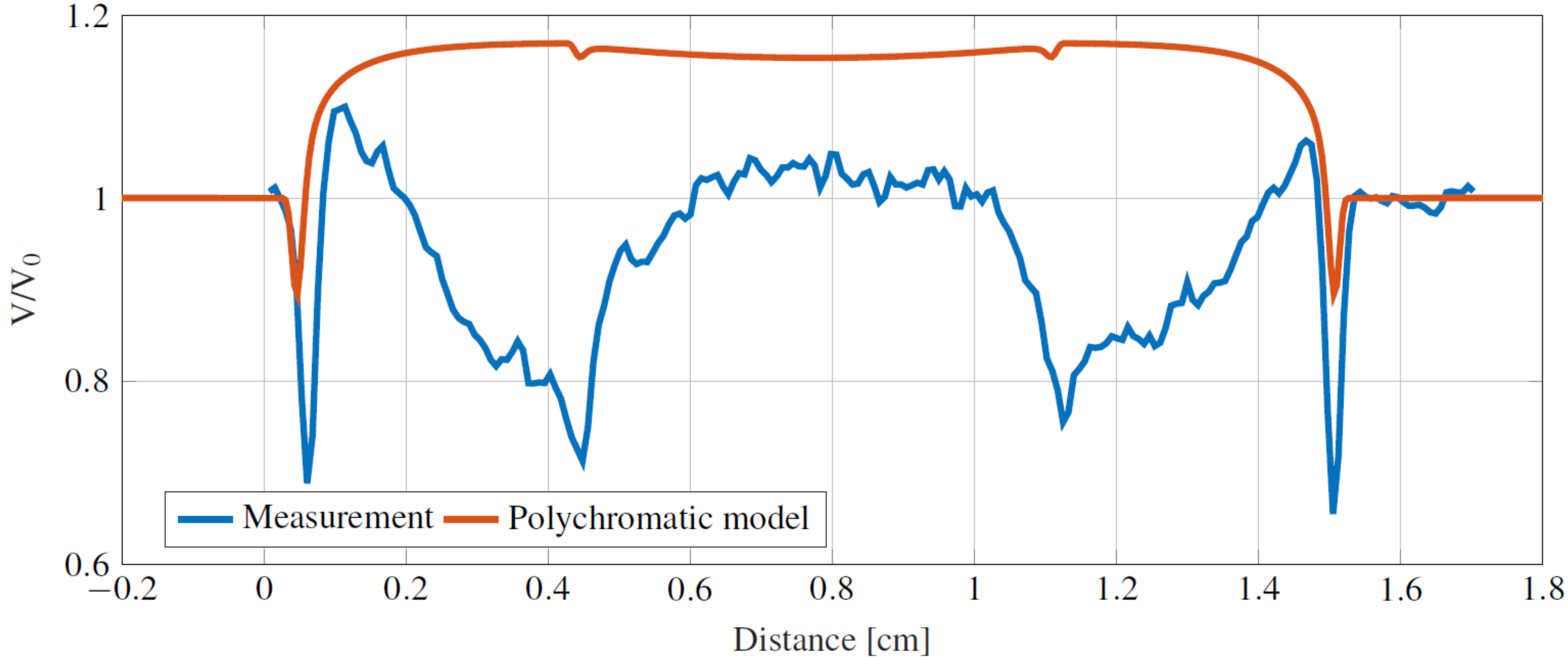
(j)

Aluminum tube at 60 kVp



(a) FBP μ (b) FBP δ (c) FBP σ

Model vs. Reality



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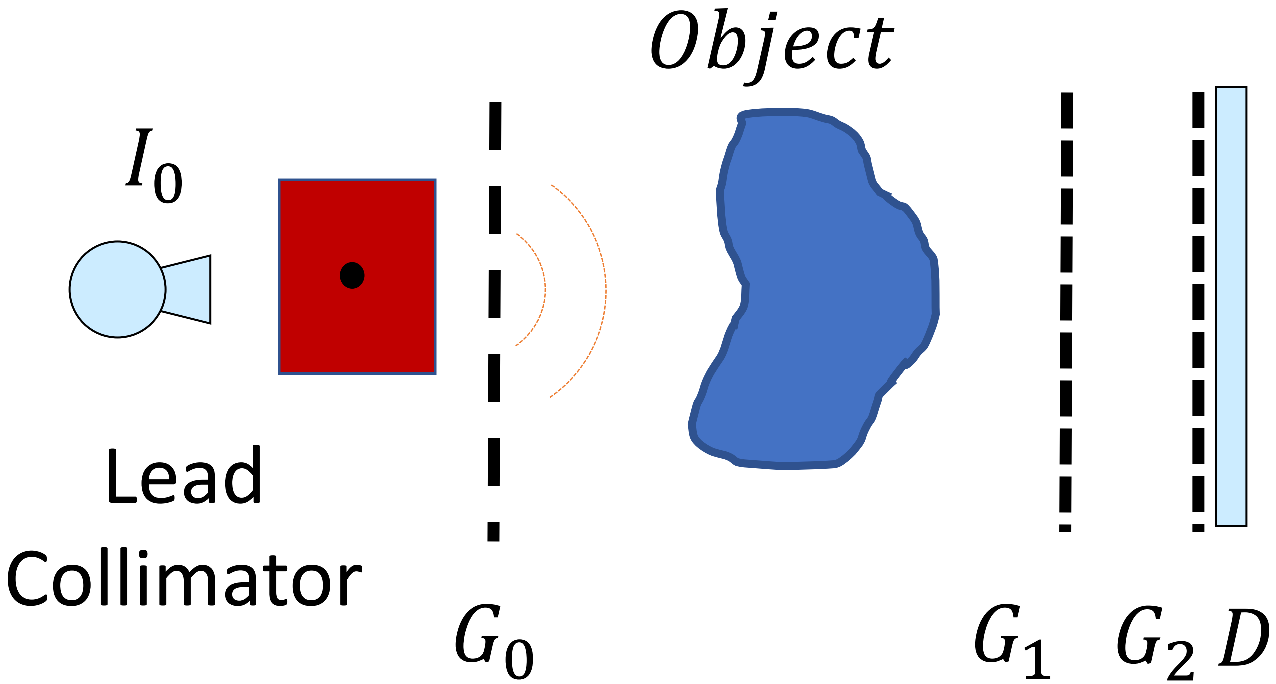
Why the discrepancy?

Compton scatter?

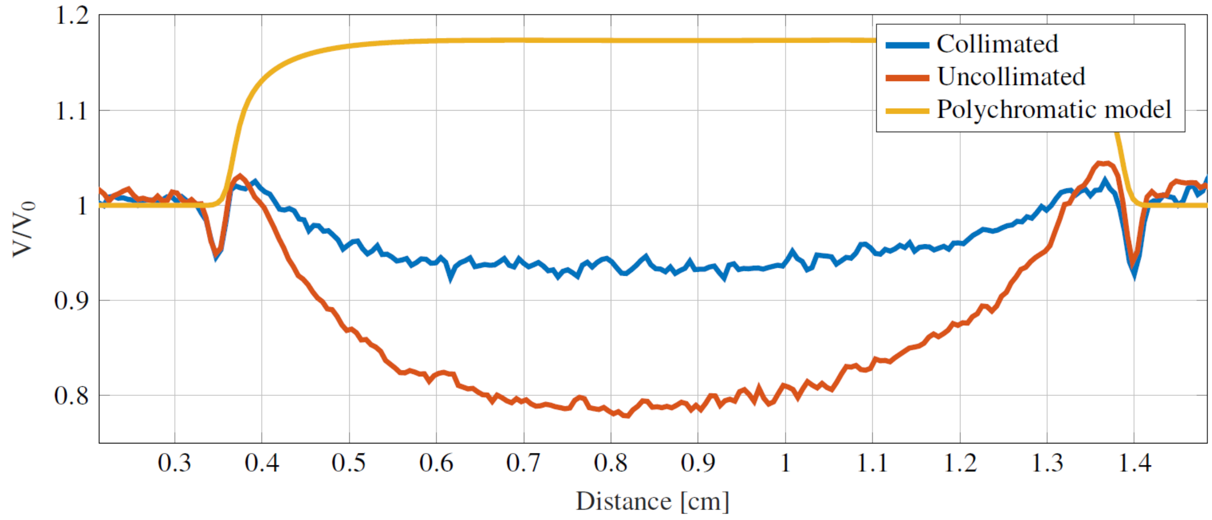
$$V = \frac{A_0 \cdot e^{-\mu \cdot d} \cdot e^{-\sigma \cdot d}}{N_0 \cdot e^{-\mu \cdot d}} = V_0 \cdot e^{-\mu \cdot d}$$

$$V' = \frac{A_0 \cdot e^{-\mu \cdot d} \cdot e^{-\sigma \cdot d}}{N_0 \cdot e^{-\mu \cdot d} + N_{Compton}}$$

Why the discrepancy?



Syringe filled with Iodine (60 kVp)



Conclusion

- Development of reconstruction framework
  - Large reconstruction resolution possible
- Development of a polychromatic forward model
  - Can reconstruct synthetic phantom data
  - Discrepancy between real and expected data

Outlook

- More evaluation on **real data**
- Adaption of the forward model

Thank you

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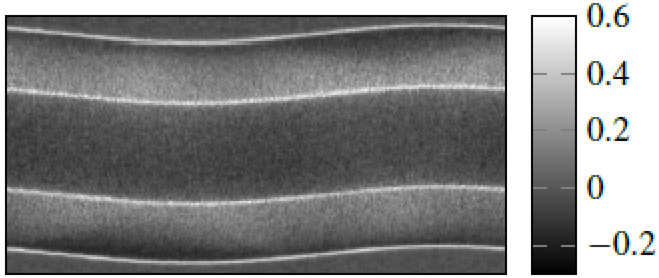
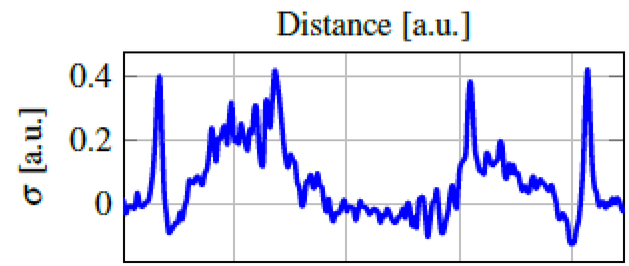
Comparison of real and simulated data (radiographic)

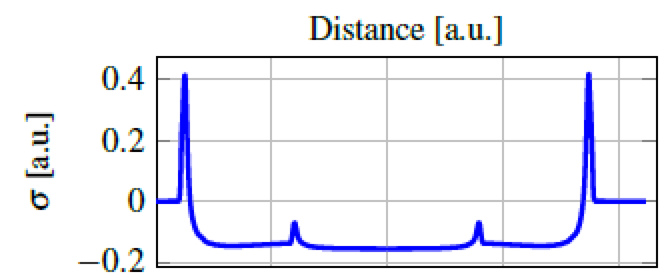


(a) Measured data

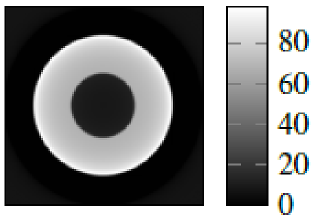


(b) Cross-section

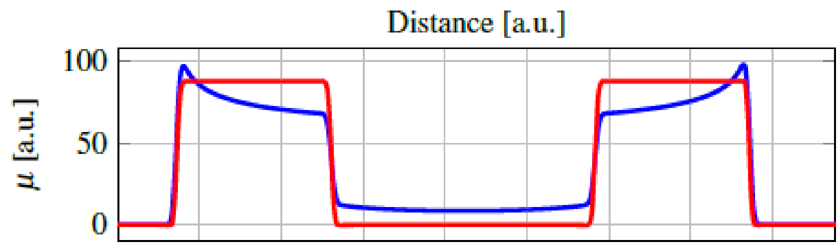
(c) Polychromatic model



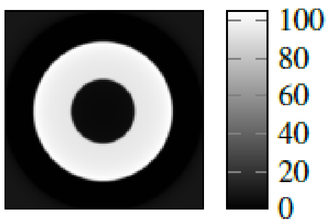
(d) Cross-section



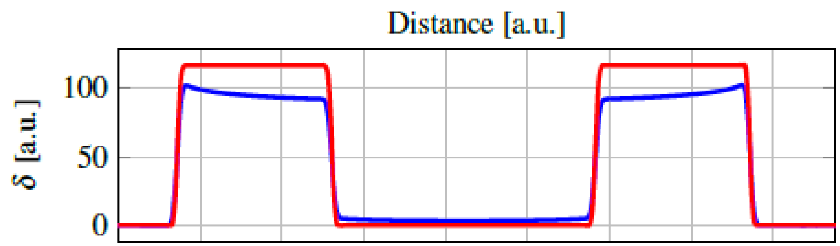
(a) Measured data



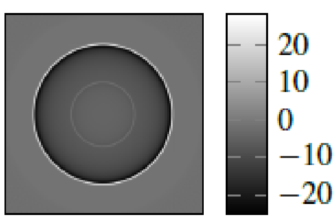
(b) Cross-section



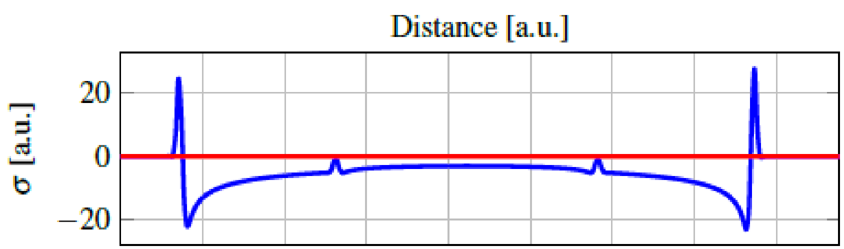
(c) Polychromatic model



(d) Cross-section



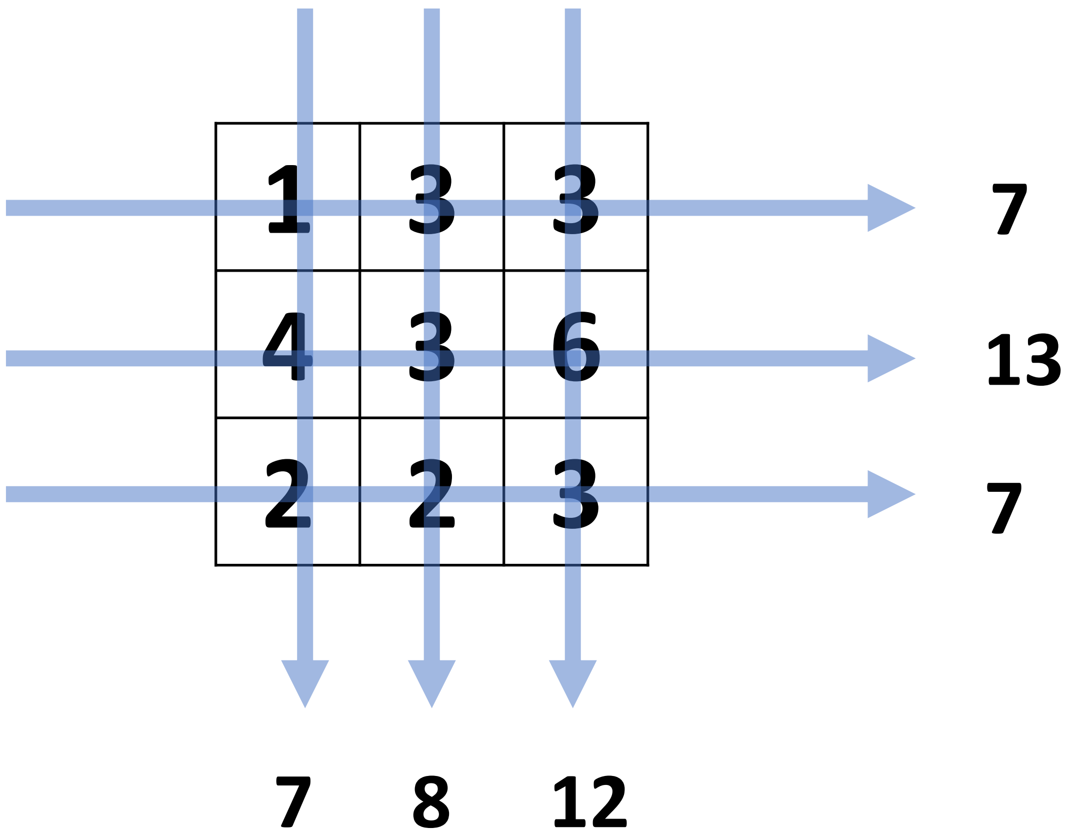
(e) Polychromatic model



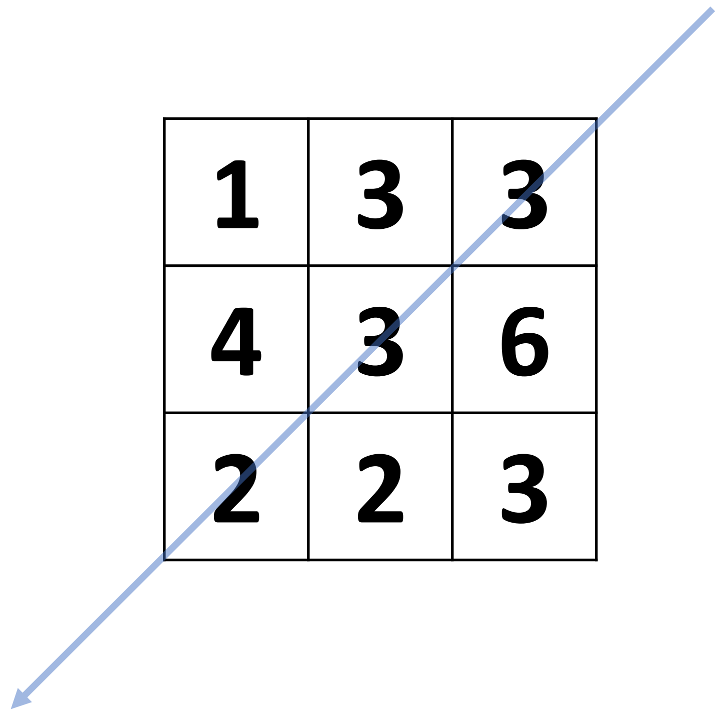
(f) Cross-section

How to calculate the line integrals?

Summation over coefficients

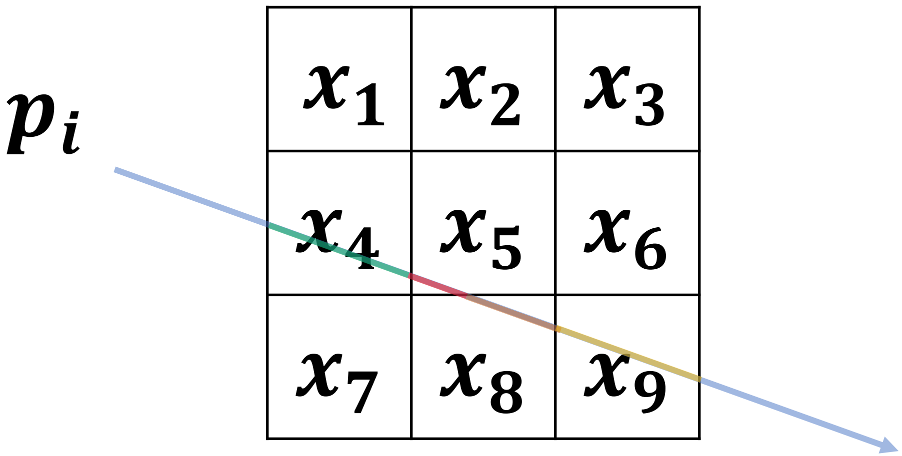


Line integral as weighted summation



$$\boldsymbol{p} = \sqrt{2} \cdot 2 + \sqrt{2} \cdot 3 + \sqrt{2} \cdot 3$$

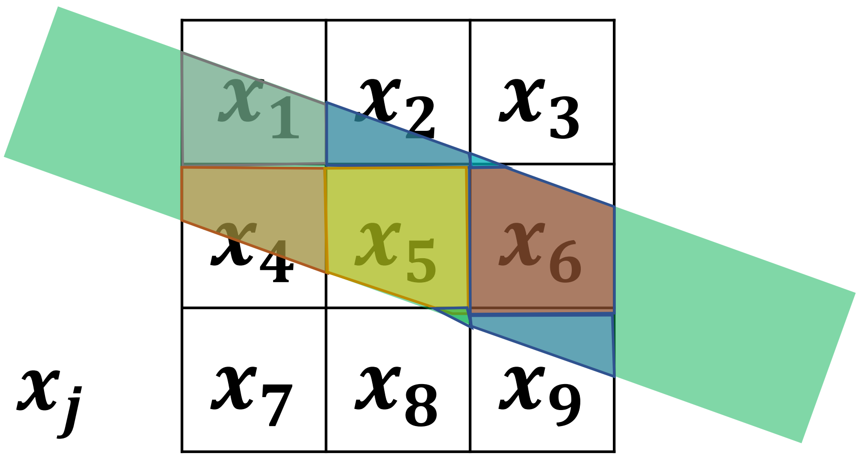
Line integral as weighted summation



$$p_i = \sum_j a_{ij} \cdot x_j$$

$$= a_4 \cdot x_4 + a_5 \cdot x_5 + a_8 \cdot x_8 + a_9 \cdot x_9$$

Line integrals as area weighted summation

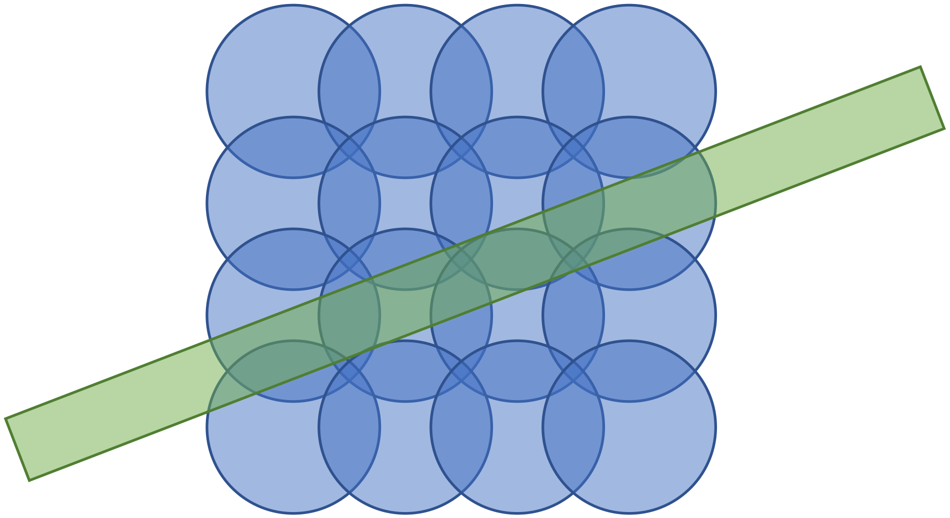


$$p_i = \sum_j a_{ij} \cdot x_j$$

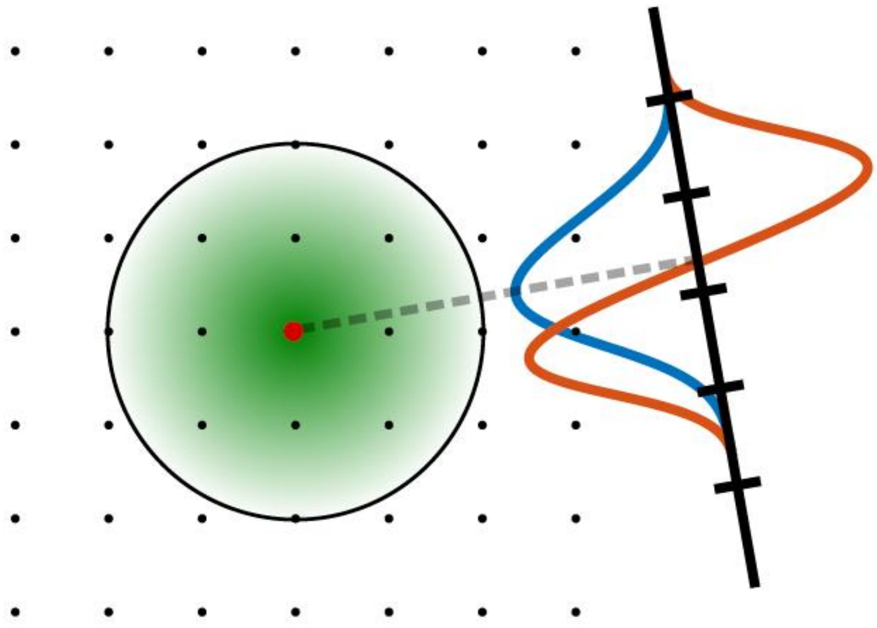
$$= a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4$$

$$+ a_5 x_5 + a_6 x_6 + a_8 x_8 + a_9 x_9$$

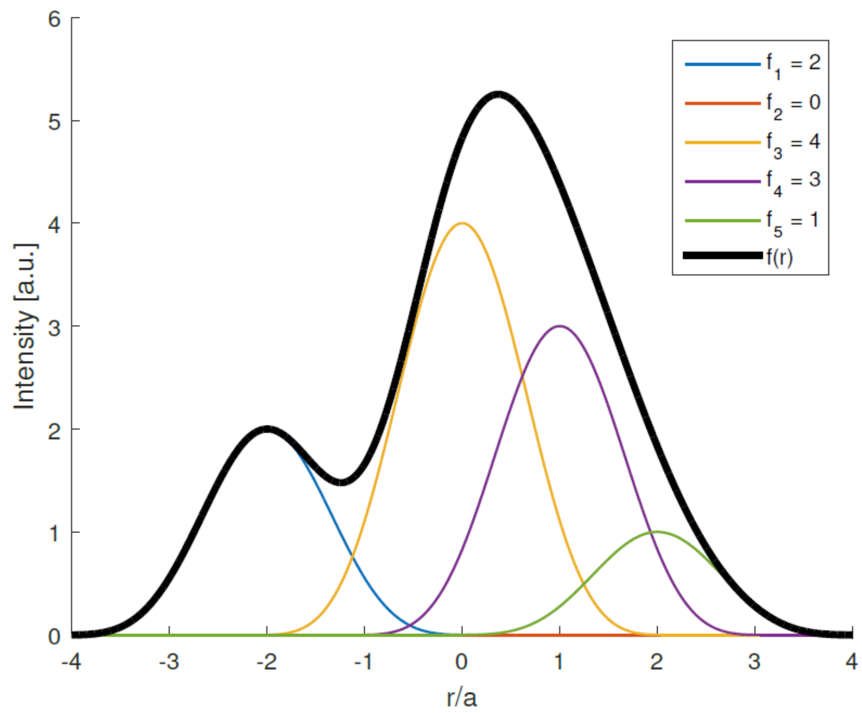
Non-rectangular representation of basis function



Non-rectangular basis function



(a) Kaiser-Bessel function and the (differential) footprint



(b) Smooth image function

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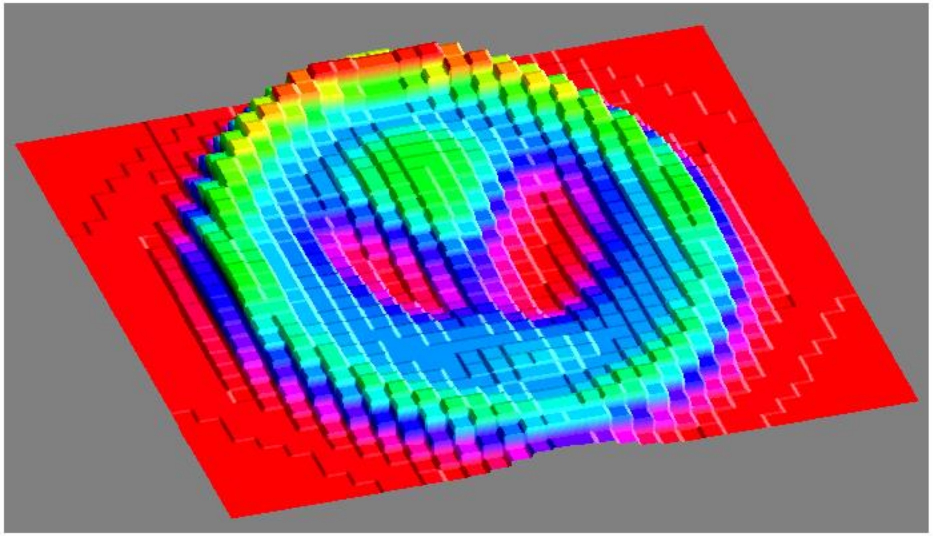
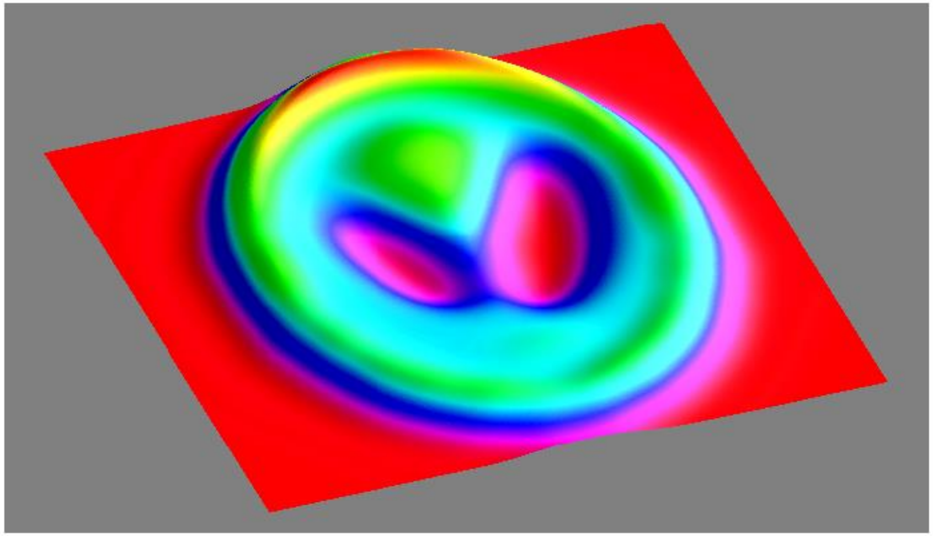
Discretization

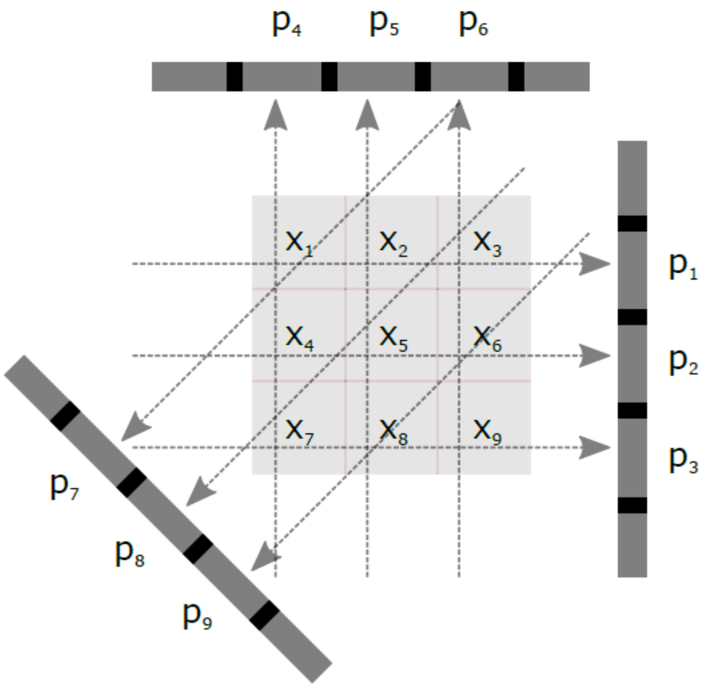


(a) Voxel discretization (32 x 32 pixel).



(b) Blob discretization (32 x 32 pixel).

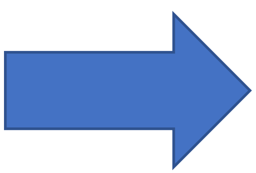
Number of matrix elements



$$N_{Total} = N_{Grid}^2 \cdot N_{Pixel} \cdot N_{Proj}$$

$$N_{Total} = 512^2 \cdot 1000 \cdot 720$$

$$= 1.8 \cdot 10^{11}$$



Memory efficient implementation

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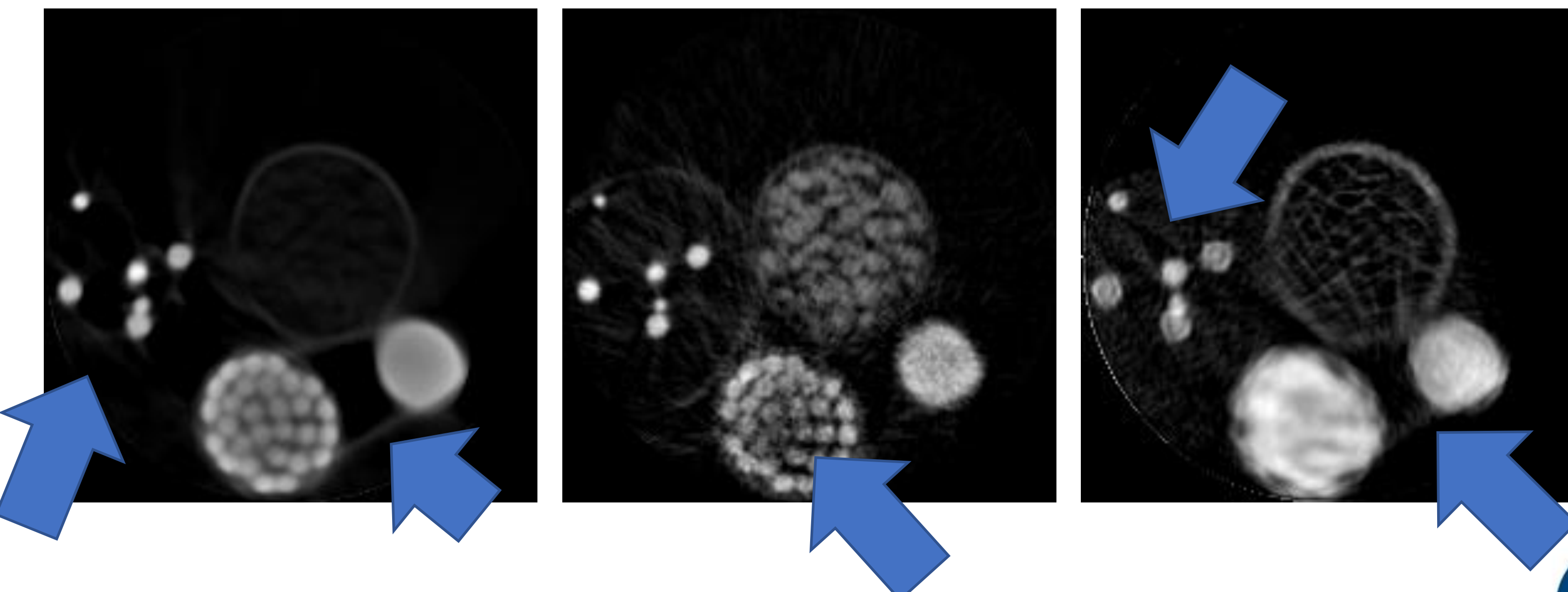
Outlook

- Evaluation on **real data**

μ δ σ



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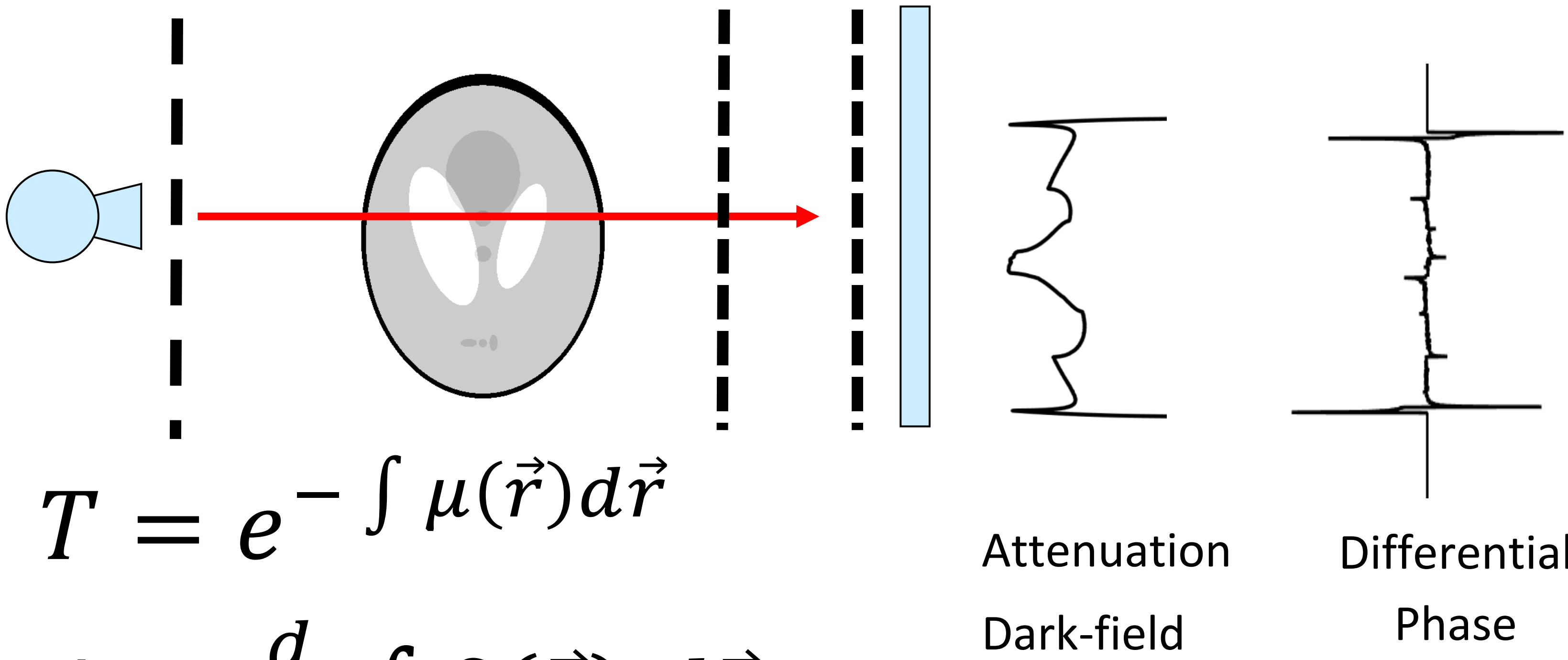
Calculating three kinds of image information

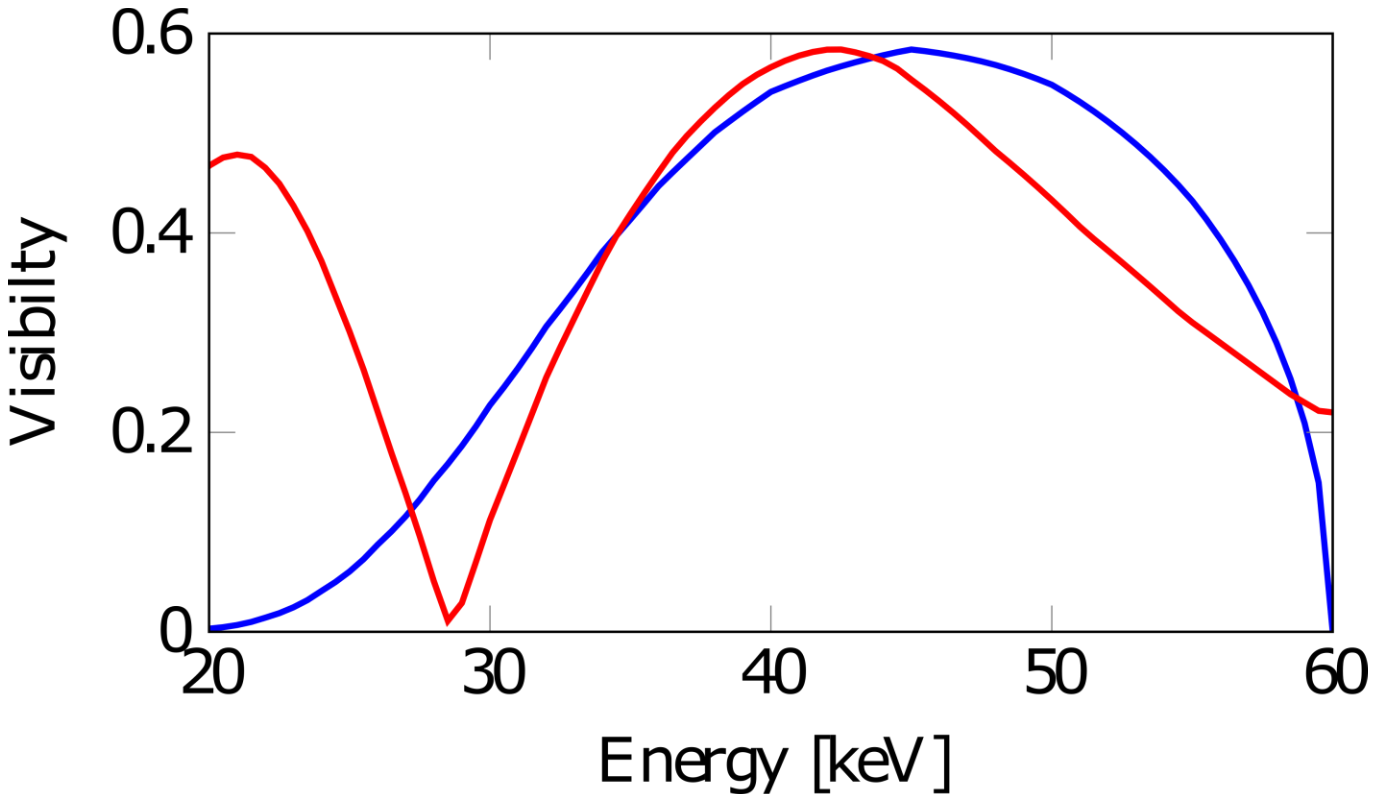


$$T = e^{-\int \mu(\vec{r})d\vec{r}}$$

$$\Delta\phi = \frac{d}{dx}\int \delta(\vec{r})\, d\vec{r}$$

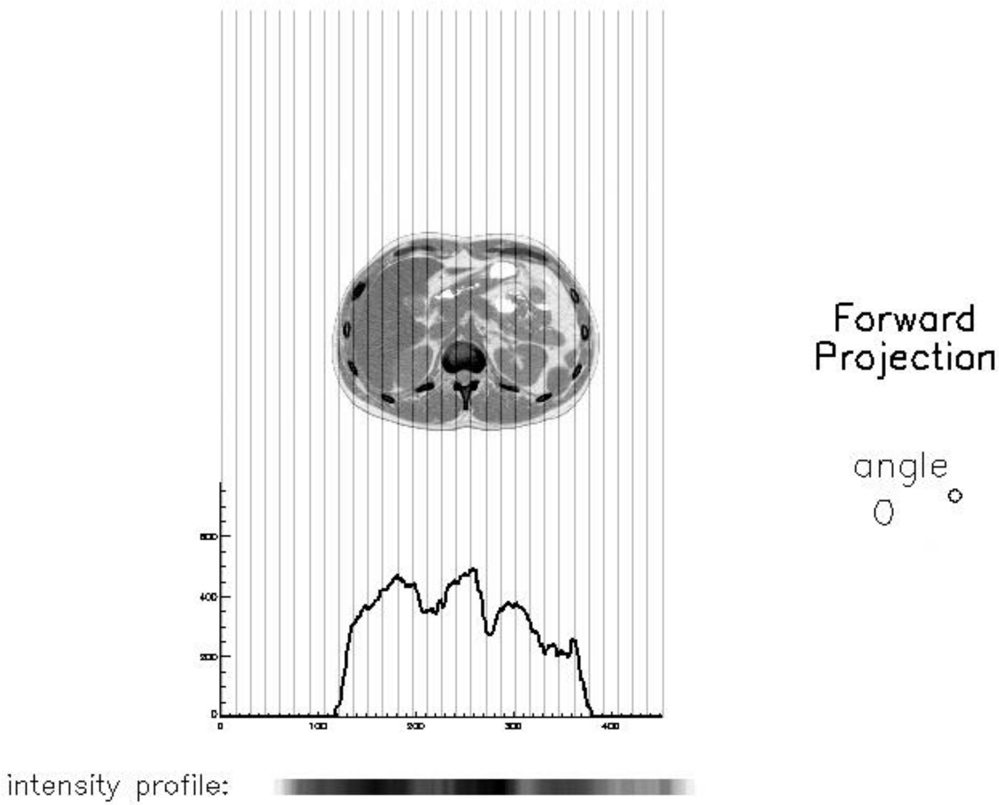
$$D = e^{-\int \sigma(\vec{r})d\vec{r}}$$

Full Setup Information



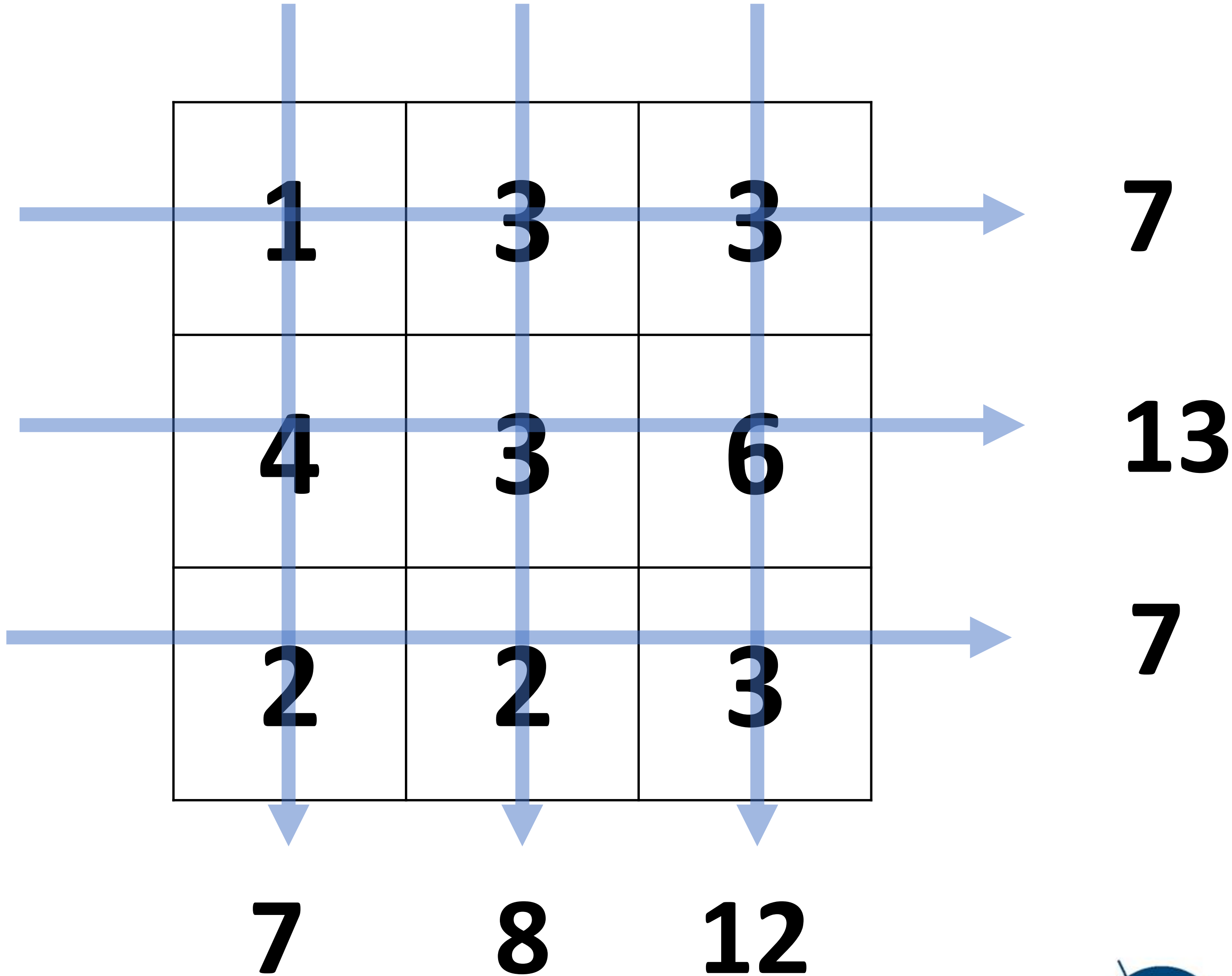
Radon transform

Sinogram



θ

detector


1 3 3
4 3 6
2 2 3
7
13
7
7 8 12